Qantum Refrigerators: The Quest to Cool to the absolute zero

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Dynamical view of the III law of thermodynamics

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Quantum thermodynamics

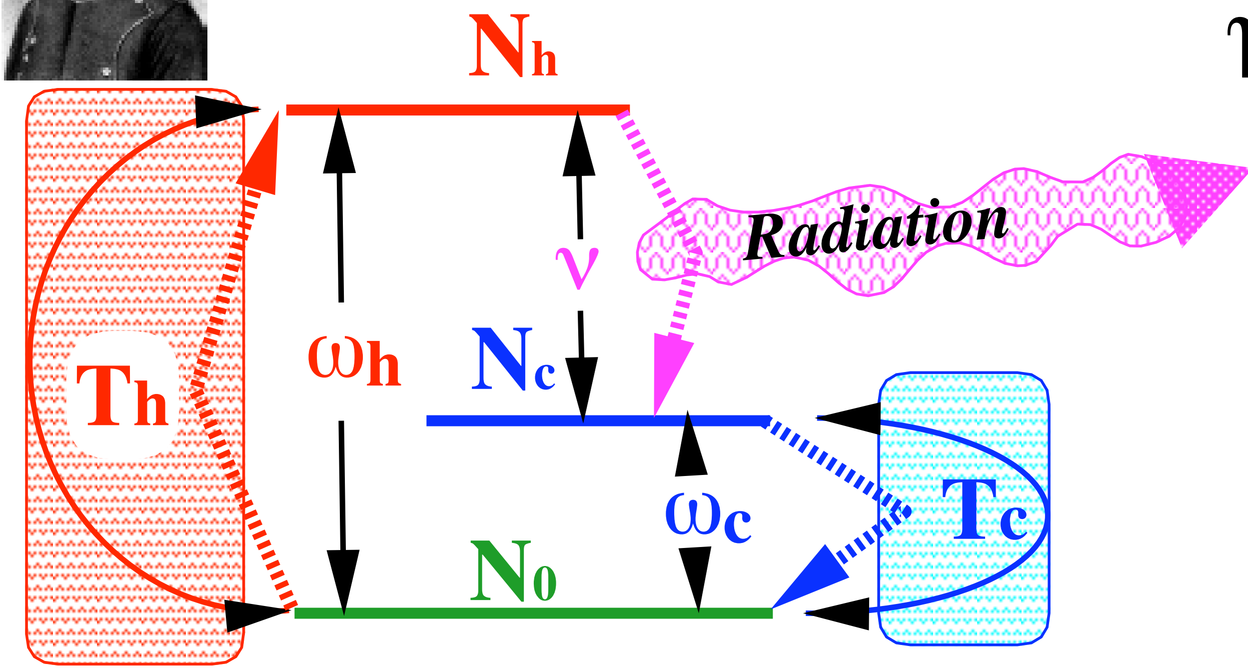
When dealing with increasing complexity we need some simple guidelines without going into the gory details.
For example no-go theorems:

A unitary evolution cannot cool a system.
A linear network cannot operate as a refrigerator or as an engine.

One cannot create a pure state.

Spontaneously we go downhill
Entropy generation should be positive.

Carnot efficiency of a 3–level amplifier



$$\eta = \frac{\nu}{\omega_h} = \frac{\omega_h - \omega_c}{\omega_h}$$

$$\boxed{G = N_h - N_c \geq 0}$$

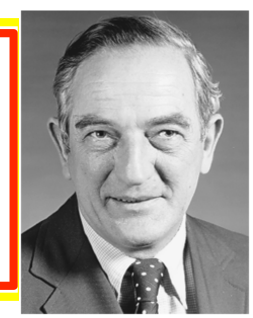
$$\frac{\hbar\omega_h}{k\,T_h} - \frac{\hbar\omega_c}{k\,T_c} \leq 0$$

$$\frac{\omega_c}{\omega_h} \leq \frac{T_c}{T_h}$$

$$N_c = N_0\, e^{-\frac{\hbar\omega_c}{k\,T_c}}$$

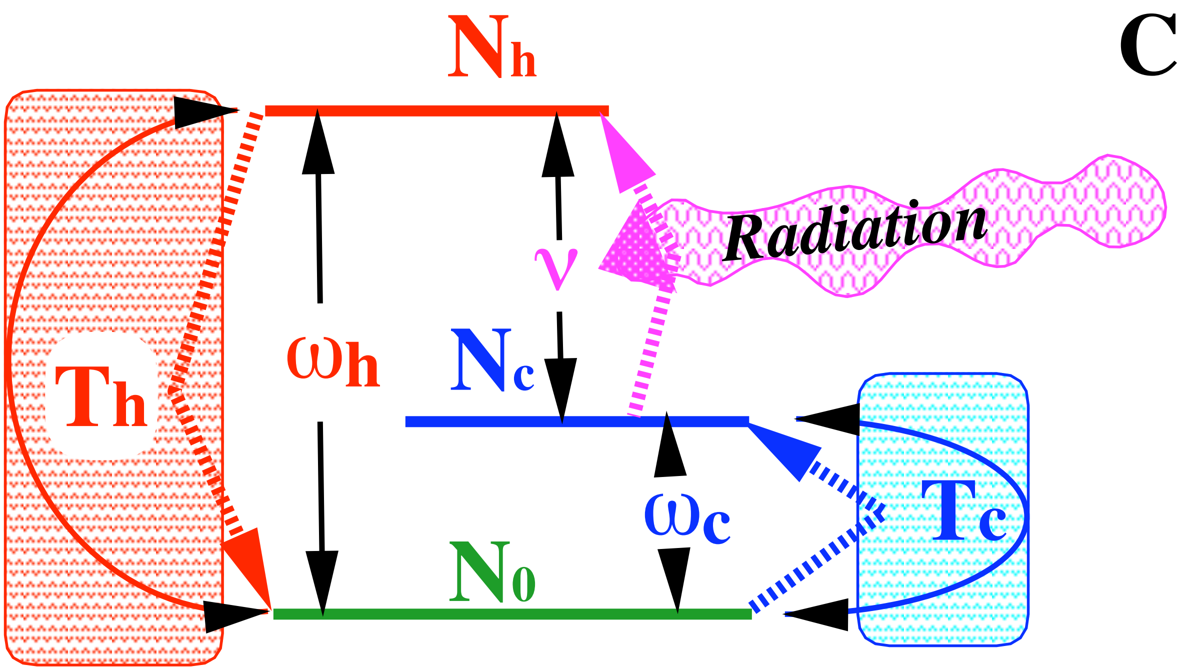
$$N_h = N_0\, e^{-\frac{\hbar\omega_h}{k\,T_h}}$$

$$\boxed{\eta = 1 - \frac{\omega_c}{\omega_h} \leq 1 - \frac{T_c}{T_h}}$$



Scovil & Shultz de Bois PRL 2 262 (1959)

Laser Cooling reversing the 3–level amplifier



$$COP = \frac{\omega_c}{\nu} = \frac{\omega_c}{\omega_h - \omega_c}$$

$$\boxed{G = N_h - N_c \leq 0}$$

$$\frac{\hbar\omega_h}{k\,T_h} - \frac{\hbar\omega_c}{k\,T_c} \leq 0$$

$$\frac{\omega_c}{\omega_h} \geq \frac{T_c}{T_h}$$

$$N_c = N_0\, e^{-\frac{\hbar\omega_c}{k\,T_c}}$$

$$\boxed{T_c > \frac{\omega_c}{\omega_h}\, T_h}$$

$$N_h = N_0\, e^{-\frac{\hbar\omega_h}{k\,T_h}}$$

Geusic J, Bois E, De Grasse R, Scovil H. J. App. Phys. 30:1113(1959)

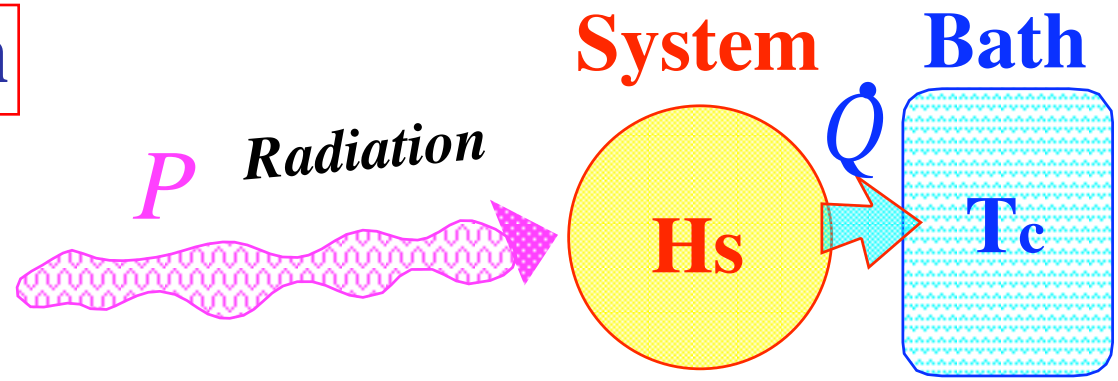
$$\boxed{COP \approx \frac{T_c}{T_h}}$$

D. J. Wineland and H. Dehmelt, Bull. Am. Phys. Soc. 20, 637 (1975); T. W. Hänsch and A. L. Schawlow, "Cooling of Gases by Laser Radiation," Opt. Commun. 13, 68 (1975).

Inserting Dynamics into Thermodynamics

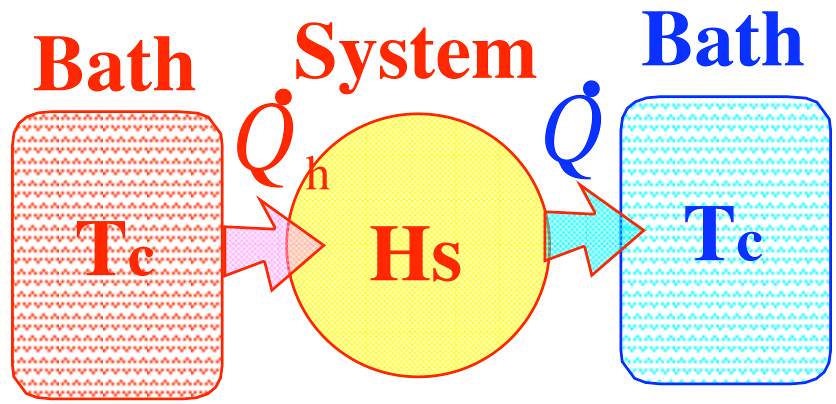
Open quantum system

$$\dot{\rho} = -i[H,\rho]+L_D(\rho)$$



$$\dot{X} = +i[H,X]+L_D^*(X)+\frac{\partial X}{\partial t}$$

Heisenberg equation of motion

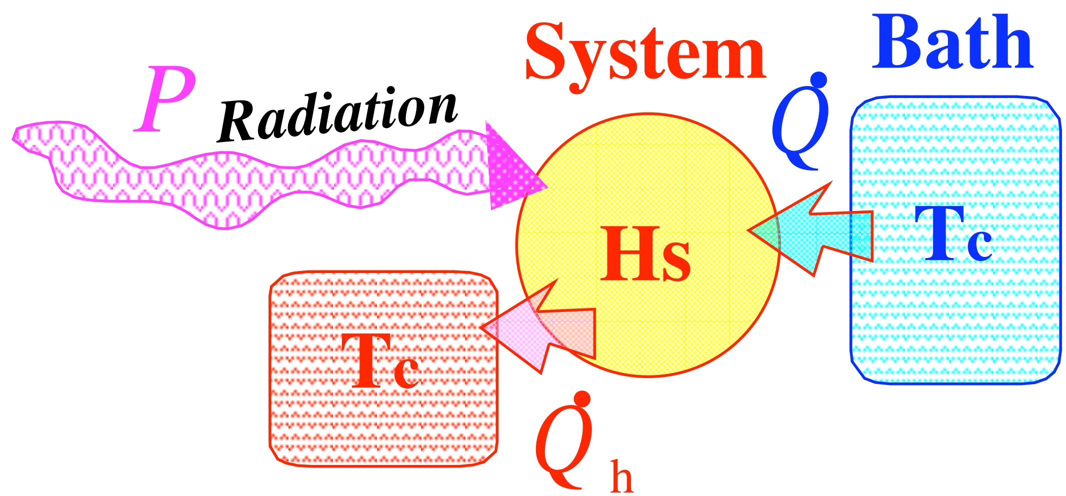


L_D is the generator of the quantum dynamical semigroup

(Quantum Master Equation)

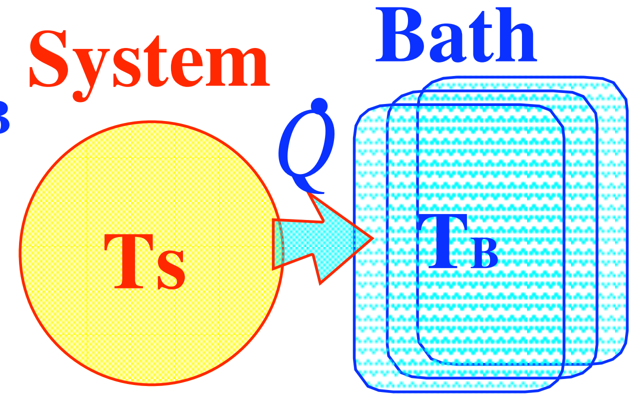
L_D Lindblad's form

$$L_D(X) = \sum_j V_j X V_j^\dagger - 1/2\{V_j V_j^\dagger, X\}$$



Inserting Dynamics into Thermodynamics

0) The zeroth law of thermodynamics: $T_S \rightarrow T_B$



Q: Isothermal partition ≡ weak coupling limit

$$\boxed{\rho_{S\otimes B} = \rho_S \otimes \rho_B}$$

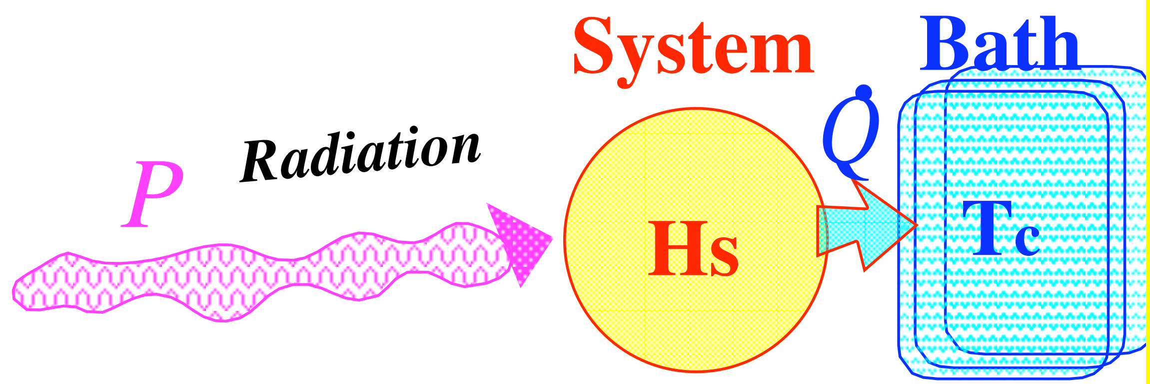
At all times.

1) Time derivative of first law of thermodynamics: energy balance

Q: Quantum definition of work and heat current

$$\dot{E} = \langle L_D(H) \rangle + \left\langle \frac{\partial H}{\partial t} \right\rangle$$

$$\dot{E} = \dot{Q} + P$$



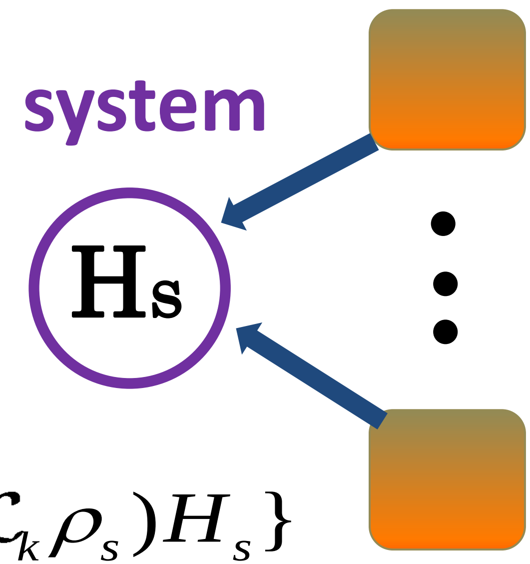
2) Second law of thermodynamics: irreversibility: work → heat

$$\boxed{\frac{d}{dt} S_S + \frac{d}{dt} S_B \geq 0}$$

$$S = -\, tr\{ \rho \ln \rho \}$$

Inserting Dynamics into Thermodynamics

Open quantum system:



Internal energy

$$\frac{dE_s}{dt} = tr\{\dot{\rho}_s H_s\} + tr\{\rho_s \dot{H}_s\}$$

Heat current

$$\mathcal{J} = tr\{\dot{\rho}_s H_s\} = \sum_k \mathcal{J}_k = \sum_k Tr\{(\mathcal{L}_k \rho_s) H_s\}$$

Power

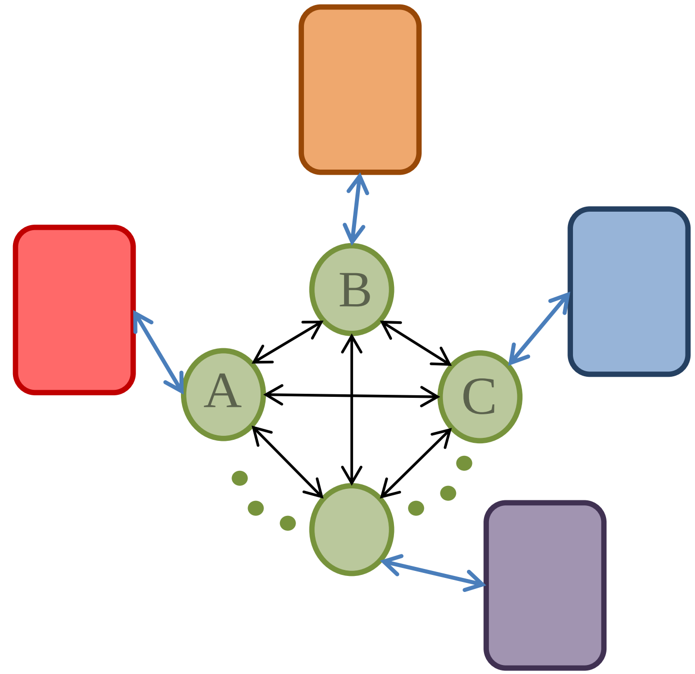
$$\mathcal{P} = tr\{\rho_s \dot{H}_s\}$$

R. Alicki , J.phys. A Math. Gen. 12 L103 (1979)

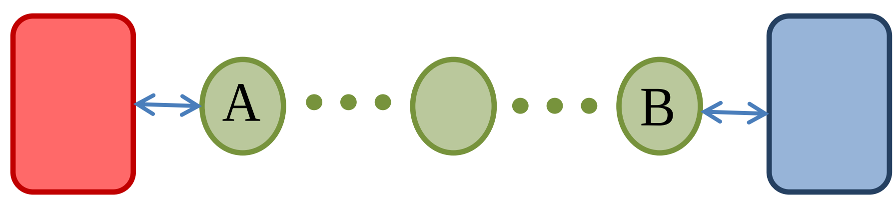
A. Levy and RK arXiv:1402.3825; EPL 107 20004 (2014)

Quantum devices

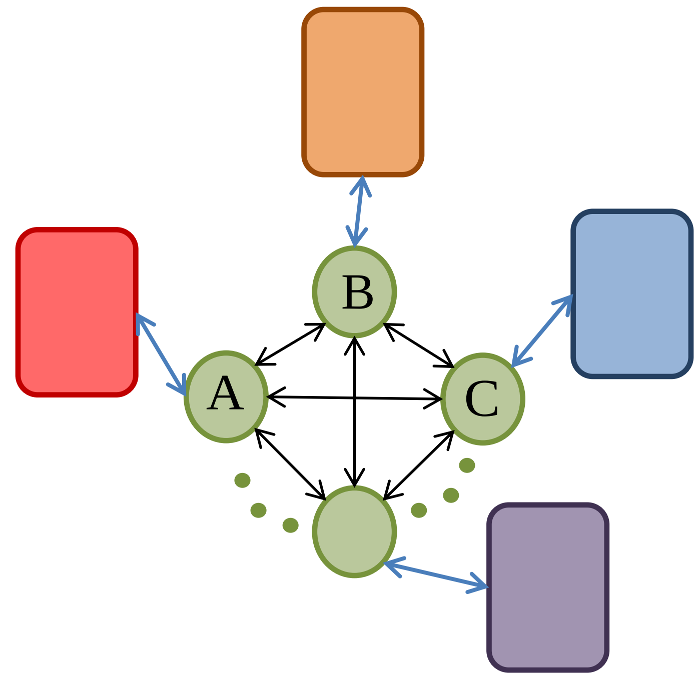
- Photo-voltaic devices
- Molecular electronics
- Quantum refrigerators
- Quantum heat engines

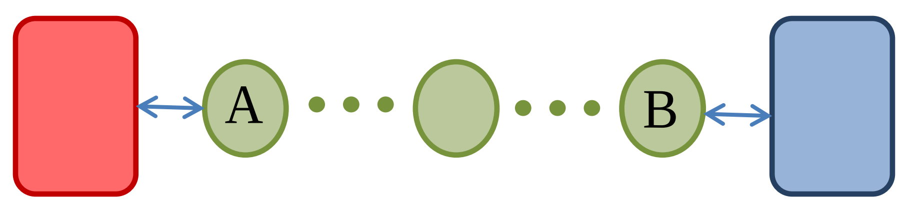


It is desired to have a framework consistent with thermodynamics



Framework

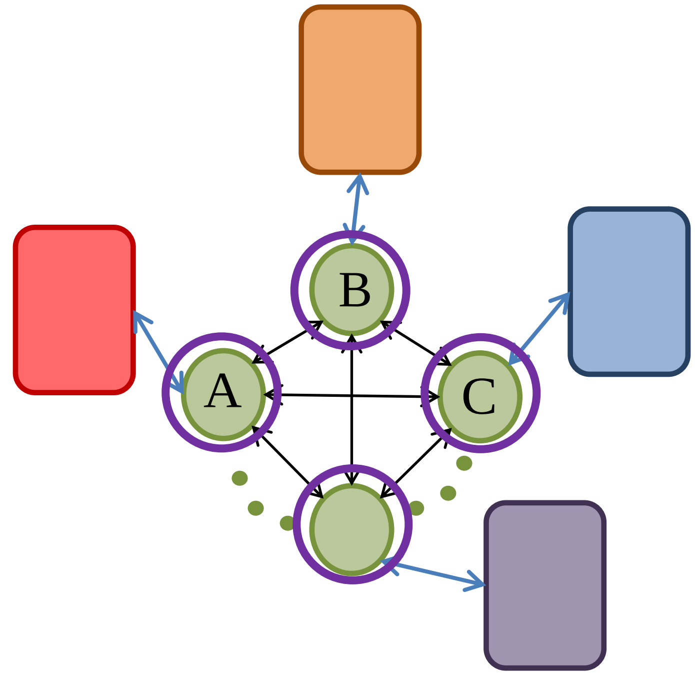


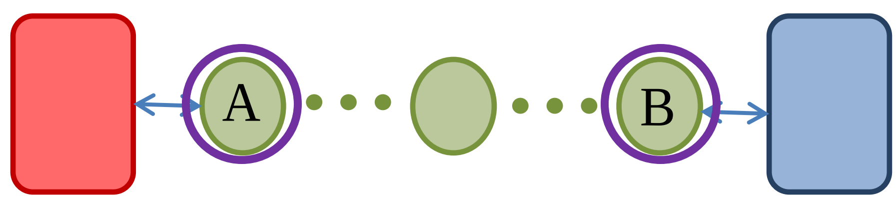


Framework

Local approach

$$\frac{d\rho_s}{dt} = -i[H_s, \rho_s] + (\mathcal{L}_A + \mathcal{L}_B + \ldots)\rho_s$$

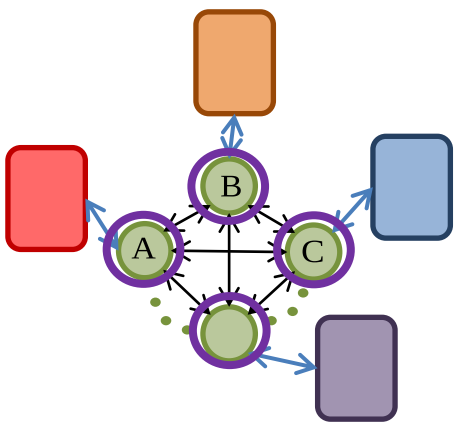


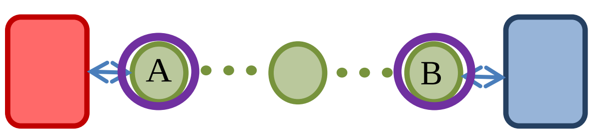


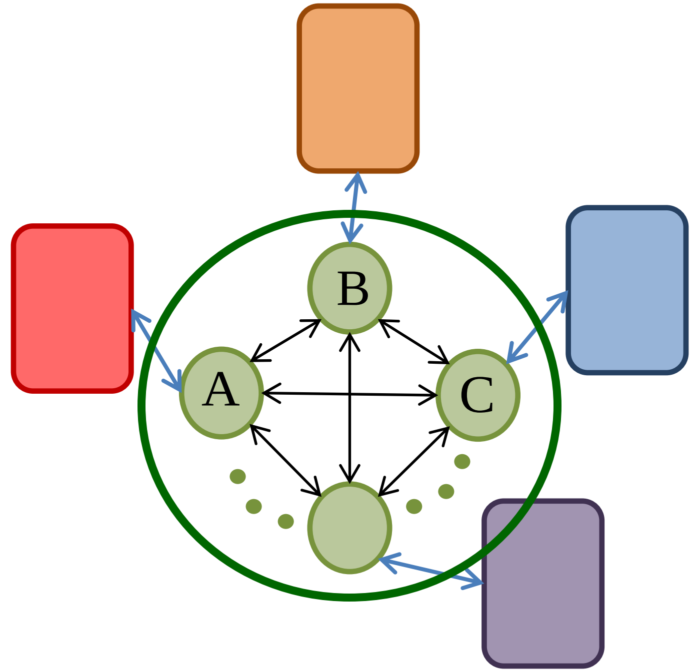
Framework

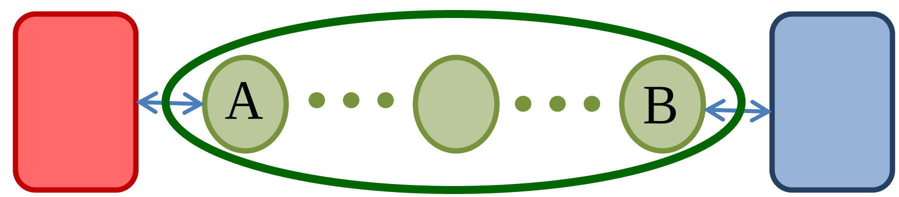
Global approach

Local approach





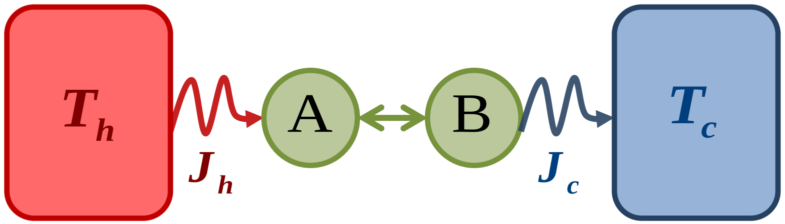




Local and *Global* approaches to quantum transport

$$\boxed{H_s = \omega_h a^\dagger a + \omega_c b^\dagger b + \varepsilon(a^\dagger b + a b^\dagger)}$$

$$H_{\text{int}} = g_{Ah}(a + a^\dagger) \otimes R_h + g_{Bc}(b + b^\dagger) \otimes R_c$$



The reduced dynamics

$$\frac{d\rho_s}{dt} = -i[H_s, \rho_s] + \mathcal{L}_h \rho_s + \mathcal{L}_c \rho_s$$

The heat flow from the hot (cold) bath

$$\mathcal{J}_{h(c)} = Tr[(\mathcal{L}_{h(c)} \rho_s) H_s]$$

A. Levy and R. Kosloff, EPL 2014 EPL 107 20004

Local and *Global* approaches to quantum transport

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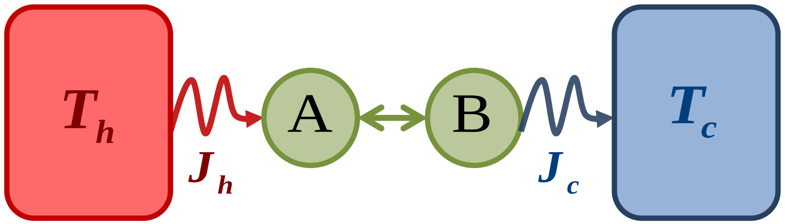

The reduced dynamics

$$\frac{d\rho_s}{dt} = -i[H_s, \rho_s] + \mathcal{L}_h \rho_s + \mathcal{L}_c \rho_s$$

The first Law:

$$\mathcal{J}_h + \mathcal{J}_c = 0$$

The heat flow from the hot (cold) bath

$$\mathcal{J}_{h(c)} = Tr[(\mathcal{L}_{h(c)} \rho_s) H_s]$$

A. Levy and R. Kosloff, EPL (accepted,2014)

Local and *Global* approaches to quantum transport

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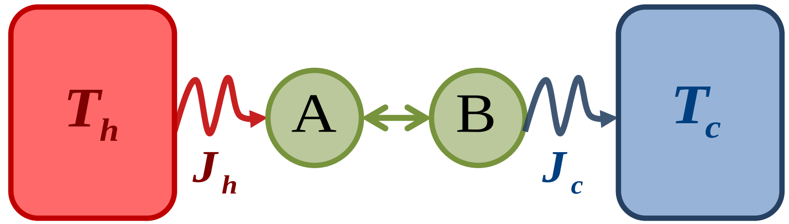


$$T_h > T_c$$

The reduced dynamics

$$\frac{d\rho_s}{dt} = -i[H_s, \rho_s] + \mathcal{L}_h \rho_s + \mathcal{L}_c \rho_s$$

The first Law:

$$\boxed{\mathcal{J}_h + \mathcal{J}_c = 0}$$

The heat flow from the hot (cold) bath

$$\mathcal{J}_{h(c)} = Tr[(\mathcal{L}_{h(c)} \rho_s) H_s]$$

The second Law:

$$-\frac{\mathcal{J}_h}{T_h} - \frac{\mathcal{J}_c}{T_c} \geq 0$$

$$\Downarrow$$

$$\mathcal{J}_h \geq 0$$

A. Levy and R. Kosloff, EPL (accepted,2014)

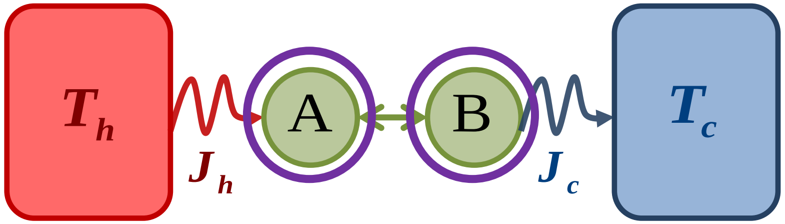
Local approach

<u>The reduced dynamics</u>

$$\frac{d\rho_s}{dt} = -i[H_s, \rho_s] + \mathcal{L}_h \rho_s + \mathcal{L}_c \rho_s$$

$$\mathcal{L}_h \rho = \frac{1}{2}\gamma_h([a, \rho a^\dagger] + e^{-\beta_h \omega_h}[a^\dagger, \rho a] + h.c.)$$

$$\mathcal{L}_c \rho = \frac{1}{2}\gamma_c([b, \rho b^\dagger] + e^{-\beta_c \omega_c}[b^\dagger, \rho b] + h.c.)$$



$$T_h > T_c$$

Local approach

The reduced dynamics

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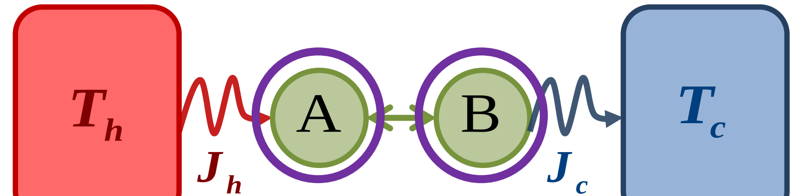

$$T_h > T_c$$

Steady state heat flow

$$\mathcal{J}_h = (e^{\beta_c \omega_c} - e^{\beta_h \omega_h})F$$

$$F > 0$$

$$\mathcal{J}_h > 0 \quad \text{for} \quad \frac{\omega_c}{T_c} > \frac{\omega_h}{T_h}$$

$$\mathcal{J}_h < 0 \quad \text{for} \quad \frac{\omega_c}{T_c} < \frac{\omega_h}{T_h}$$

Local approach

The reduced dynamics

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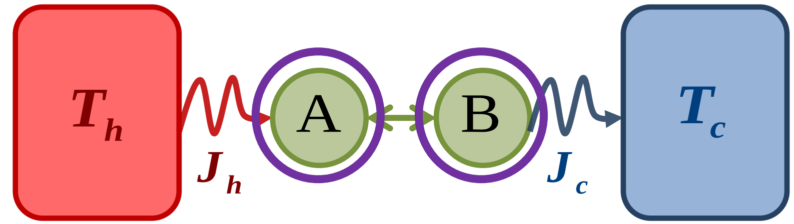

$$T_h > T_c$$

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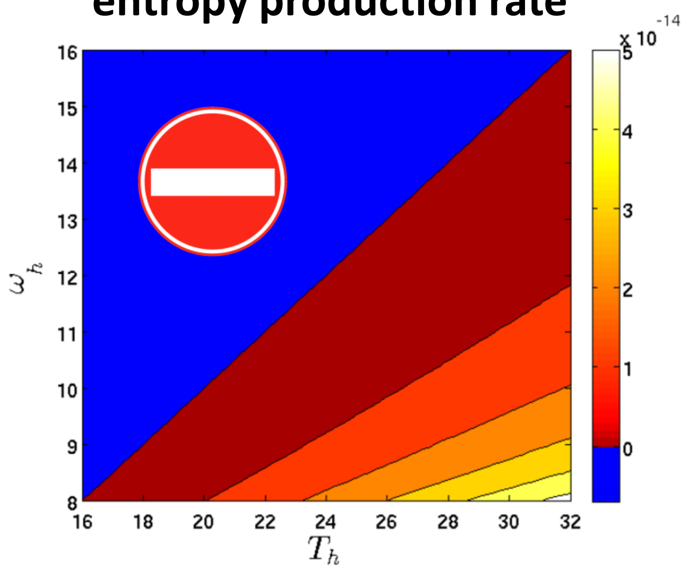
$$\mathcal{J}_h > 0 \quad \text{for} \quad \frac{\omega_c}{T_c} > \frac{\omega_h}{T_h}$$

$$\boxed{\mathcal{J}_h < 0 \quad \text{for} \quad \frac{\omega_c}{T_c} < \frac{\omega_h}{T_h}}$$

Violation of the second law!!!

entropy production rate

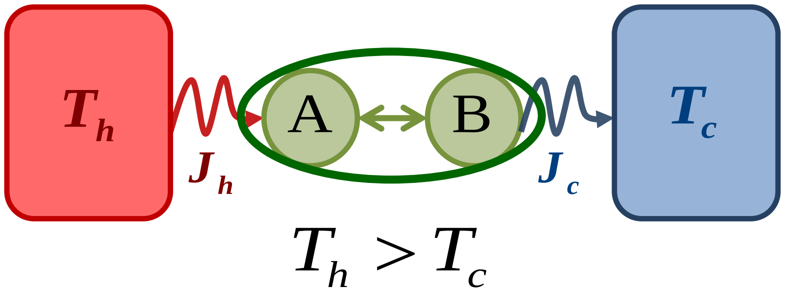


A. Levy and R. Kosloff, EPL (accepted,2014)

Global approach

The reduced dynamics

$$\frac{d\rho_s}{dt} = -i[H_s, \rho_s] + \mathcal{L}_h \rho_s + \mathcal{L}_c \rho_s$$



$$H_s = \omega_+ d_+^\dagger d_+ + \omega_- d_-^\dagger d_-$$

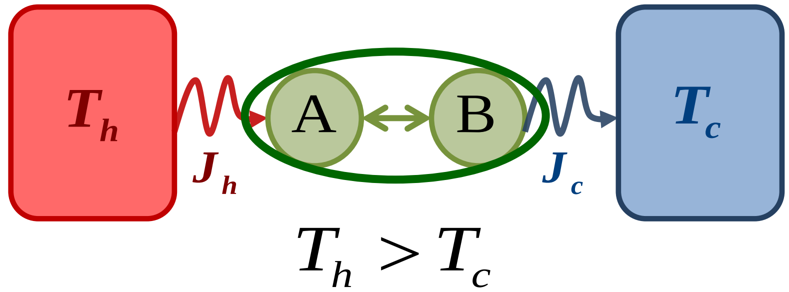
$$\omega_\pm = \frac{\omega_h + \omega_c}{2} \pm \sqrt{\left(\frac{\omega_h - \omega_c}{2}\right)^2 + \varepsilon^2}$$

$$d_+ = a\cos(\theta) + b\sin(\theta)$$
$$d_- = b\cos(\theta) - a\sin(\theta)$$

Global approach

The reduced dynamics

$$\frac{d\rho_s}{dt} = -i[H_s, \rho_s] + \mathcal{L}_h \rho_s + \mathcal{L}_c \rho_s$$



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$$H_{\text{int}} = g_{Ah}(a + a^\dagger) \otimes R_h + g_{Bc}(b + b^\dagger) \otimes R_c$$

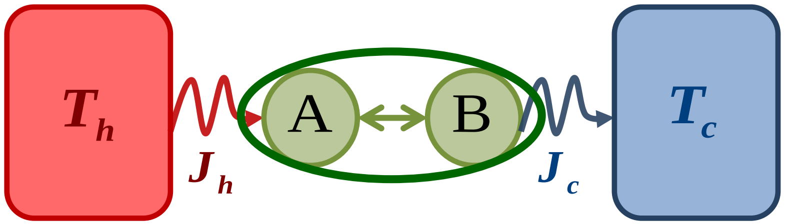
$$\Downarrow$$

$$H_{\text{int}} = g_{Ah}\left((d_+ + d_+^\dagger)\cos(\theta) - (d_- + d_-^\dagger)\sin(\theta)\right) \otimes R_h$$
$$+ g_{Bc}\left((d_+ + d_+^\dagger)\sin(\theta) + (d_- + d_-^\dagger)\cos(\theta)\right) \otimes R_c$$

Global approach

The reduced dynamics

$$\frac{d\rho_s}{dt} = -i[H_s, \rho_s] + \mathcal{L}_h \rho_s + \mathcal{L}_c \rho_s$$



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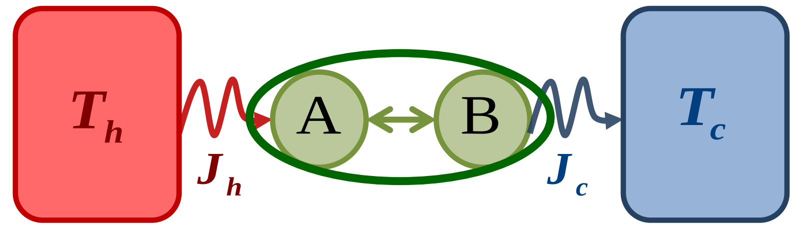
$$+ \frac{\sin^2(\theta)}{2}\gamma_h^{(-)}([d_-, \rho d_-^\dagger] + e^{-\beta_h \omega_-}[d_-^\dagger, \rho d_-] + h.c.)$$

$$\mathcal{L}_c \rho = \frac{\sin^2(\theta)}{2}\gamma_c^{(+)}([d_+, \rho d_+^\dagger] + e^{-\beta_c \omega_+}[d_+^\dagger, \rho d_+] + h.c.)$$
$$+ \frac{\cos^2(\theta)}{2}\gamma_c^{(-)}([d_-, \rho d_-^\dagger] + e^{-\beta_c \omega_-}[d_-^\dagger, \rho d_-] + h.c.)$$

Global approach

The reduced dynamics

$$\frac{d\rho_s}{dt} = -i[H_s, \rho_s] + \mathcal{L}_h \rho_s + \mathcal{L}_c \rho_s$$



$$T_h > T_c$$

$$H_s = \omega_+ d_+^\dagger d_+ + \omega_- d_-^\dagger d_-$$

$$\boxed{\mathcal{J}_h > 0}$$

Always!!!

$$\omega_\pm = \frac{\omega_h + \omega_c}{2} \pm \sqrt{\left(\frac{\omega_h - \omega_c}{2}\right)^2 + \varepsilon^2}$$

$$d_+ = a\cos(\theta) + b\sin(\theta)$$
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Global approach

The reduced dynamics

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$$H_s = \omega_+ d_+^\dagger d_+ + \omega_- d_-^\dagger d_-$$

$$\omega_\pm = \frac{\omega_h + \omega_c}{2} \pm \sqrt{\left(\frac{\omega_h - \omega_c}{2}\right)^2 + \varepsilon^2} \qquad \begin{aligned} d_+ &= a\cos(\theta) + b\sin(\theta) \\ d_- &= b\cos(\theta) - a\sin(\theta) \end{aligned}$$

$$\mathcal{L}_h \rho = \boxed{\frac{\cos^2(\theta)}{2}\gamma_h^{(+)}([d_+, \rho d_+^\dagger] + e^{-\beta_h \omega_+}[d_+^\dagger, \rho d_+] + h.c.)}$$
$$\boxed{+\frac{\sin^2(\theta)}{2}\gamma_h^{(-)}([d_-, \rho d_-^\dagger] + e^{-\beta_h \omega_-}[d_-^\dagger, \rho d_-] + h.c.)}$$

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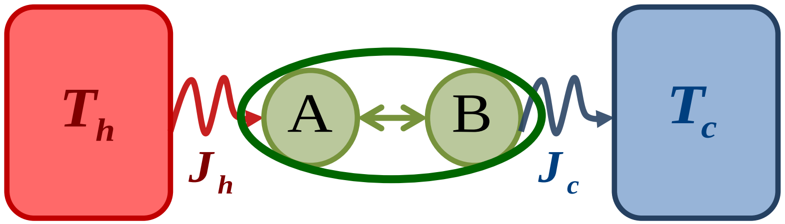


$$T_h > T_c$$

$$\boxed{\mathcal{J}_h > 0}$$

Always!!!

Spohn's inequality

$$\sigma(\rho(t)) = -Tr\left[\mathcal{L}\rho(\ln(\rho) - \ln(\tilde{\rho}))\right] \geq 0$$

$\tilde{\rho}$ - Stationary state

H. Spohn, J. Math. Phys. 19, 1227 (1978)
A. Levy and R. Kosloff, EPL (accepted,2014)

Local and *Global* approaches to quantum transport

Global

$$\boxed{\mathcal{J}_h > 0}$$

Local

$$\mathcal{J}_h = \left(e^{\beta_c\omega_c} - e^{\beta_h\omega_h}\right)F$$

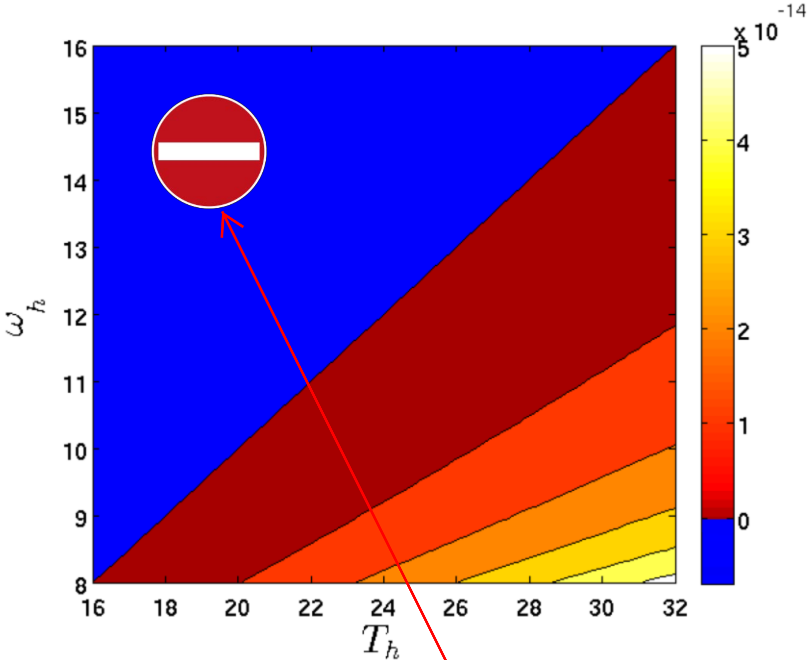
$$F > 0$$

$$\Downarrow$$

$$\mathcal{J}_h > 0 \quad \text{for} \quad \frac{\omega_c}{T_c} > \frac{\omega_h}{T_h}$$

$$\mathcal{J}_h < 0 \quad \text{for} \quad \frac{\omega_c}{T_c} < \frac{\omega_h}{T_h}$$

Rate of entropy production Local approach



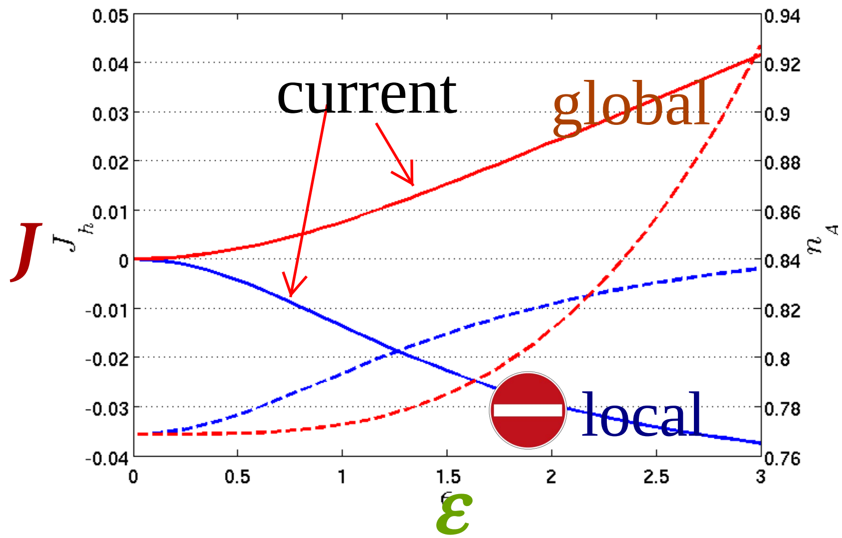
Violation of the second law!!!

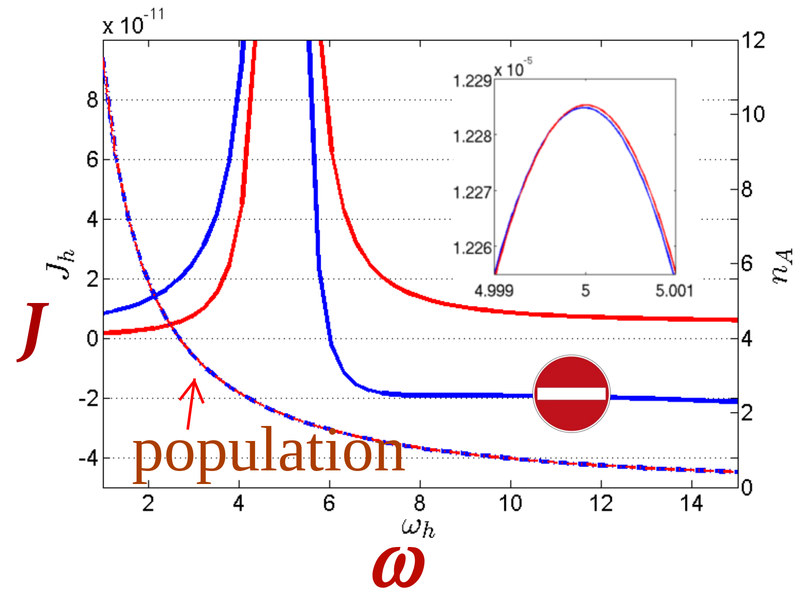
Local and *Global* approaches to quantum transport

- Local approach is incorrect for strong coupling between the junction subsystems.

- In the weak coupling limit $\varepsilon < \sqrt{|\omega_h - \omega_c|}$ local observables converge to the global result.

- Non-local observables are off for all ε . (and may lead to violation of the second law).





The third law of thermodynamics

Two independent formulations of the third-law of thermodynamics both stated by Nernst.

The first known as the "Nernst heat theorem", phrased:

- The entropy of any pure substance in thermodynamic equilibrium approaches zero as the temperature approaches zero.

The second formulation is known as the unattainability principle:

- It is impossible by any procedure, no matter how idealised, to reduce any assembly to absolute zero temperature in a finite number of operations.

Nernst heat theorem and the II-law

At steady state the II-law implies:

$$\frac{d}{dt}\Delta S^u = -\frac{\mathcal{J}_c}{T_c} - \frac{\mathcal{J}_h}{T_h} - \frac{\mathcal{J}_w}{T_w} \geq 0 \quad .$$

As $T_c \to 0$ the cold current should scale with temperature as:

$$\mathcal{J}_c \propto T_c^{1+\alpha} \quad \textit{with an exponent} \quad \alpha \ .$$

The II-law implies when $T_c \to 0$: $\Delta \dot{S}_c \sim -T_c^{\alpha} \quad , \quad \alpha \geq 0 \quad .$

For the case when $\alpha = 0$ the fulfilment of the second law depends on the entropy production of the other baths $-\frac{\mathcal{J}_h}{T_h} - \frac{\mathcal{J}_w}{T_w} > 0$
Nernst's heat theorem then leads to the scaling condition :

$$\boxed{\mathcal{J}_c \sim T_c^{\alpha+1} \quad \textit{and} \quad \alpha > 0 \ .} \tag{1}$$

The unattainability principle.

A dynamical interpretation of the III-law:

No refrigerator can cool a system to absolute zero temperature at finite time.

The unattainability principle is quantified by the characteristic exponent ζ:

$$\boxed{\frac{dT_c(t)}{dt} = -c\; T_c^{\zeta}, \quad T_c \rightarrow 0 \;\; .} \qquad (2)$$

where c is a positive constant. Solving Eq. (2), leads to:

$$T_c(t)^{1-\zeta} = T_c(0)^{1-\zeta} \; - ct \qquad , \; for\; \zeta < 1 \;\; ,$$

$$T_c(t) \;=\; T_c(0)e^{-ct} \qquad , \; for\; \zeta = 1 \;\; ,$$

$$\frac{1}{T_c(t)^{\zeta-1}} = \frac{1}{T_c(0)^{\zeta-1}} \quad + ct \qquad , \; for\; \zeta > 1 \;\; ,$$

Reaching zero temperature in finite time!

The two third-law scaling relations can be related by accounting for the heat capacity $c_V(T_c)$ of the cold bath:

$$\mathscr{J}_c(T_c(t)) = -c_V(T_c(t))\frac{dT_c(t)}{dt} \quad .$$

$c_V(T_c)$ is determined by the behaviour of the degrees of freedom of the cold bath at low temperature where $c_V \sim T_c^{\eta}$ when $T_c \to 0$. Therefore the scaling exponents are related

$$\zeta = 1 + \alpha - \eta$$

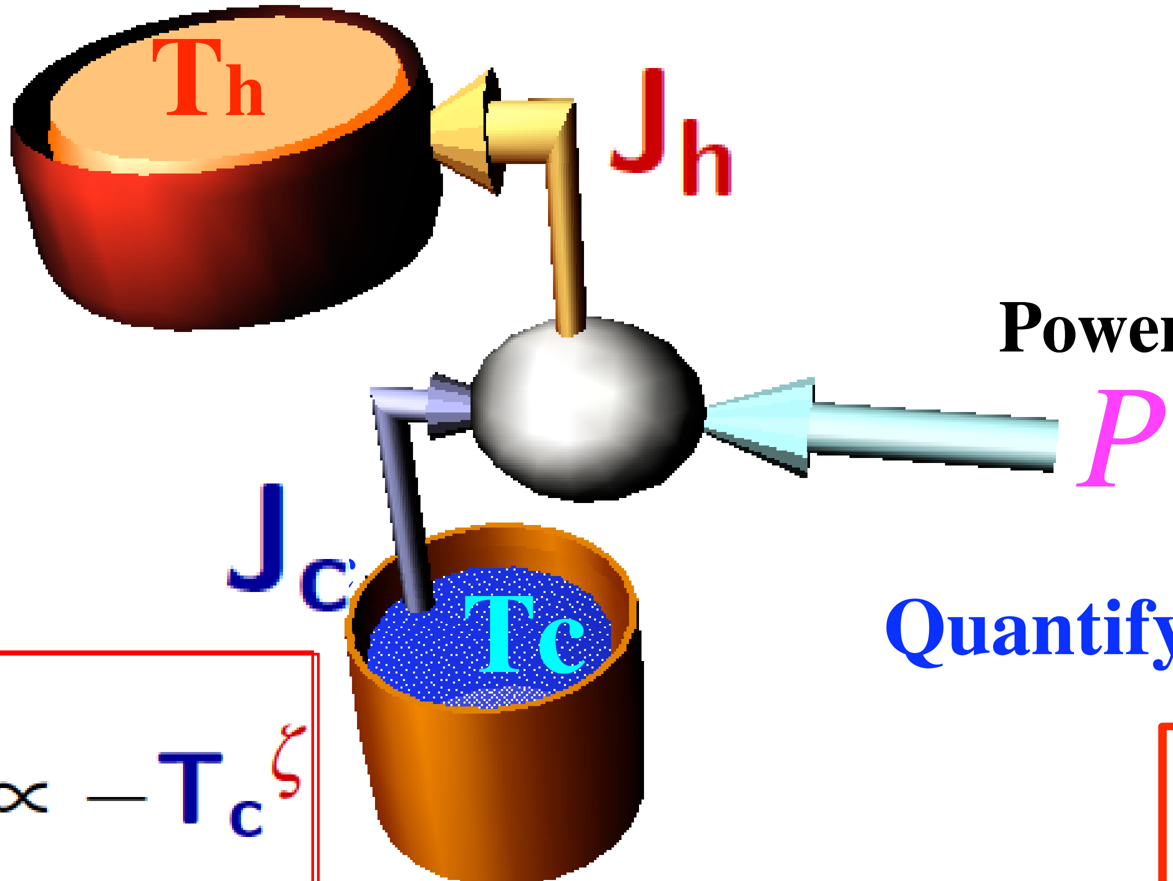
.

When $T_c \to 0$ the cold bath state ρ_c reaches zero entropy it becomes pure, therefore it is unentangled with the environment.

$$\hat{\rho} = \hat{\rho}_c \otimes \hat{\rho}_B$$

The quest to cool to the absolute zero temperature

hot reservoir



cold reservoir

$$\frac{dT_c}{dt} \propto -{T_c}^{\zeta}$$

what is the exponent ζ when $T_c \to 0$.

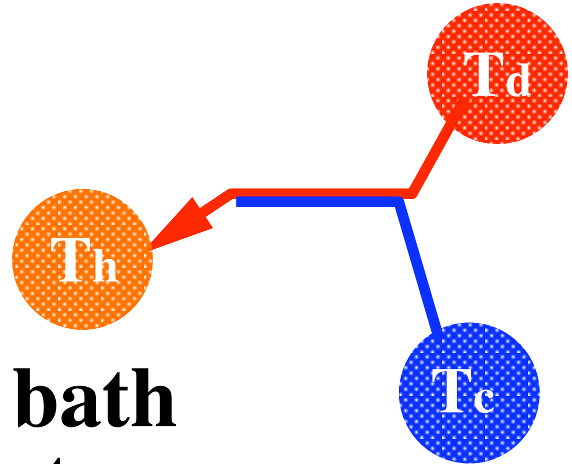
Quantifying the third law

$$\frac{dJ_c}{dt} \propto -{T_c}^{\alpha+1}$$

what is α when $T_c \to 0$

Absorption refrigerator

Using heat to cool!

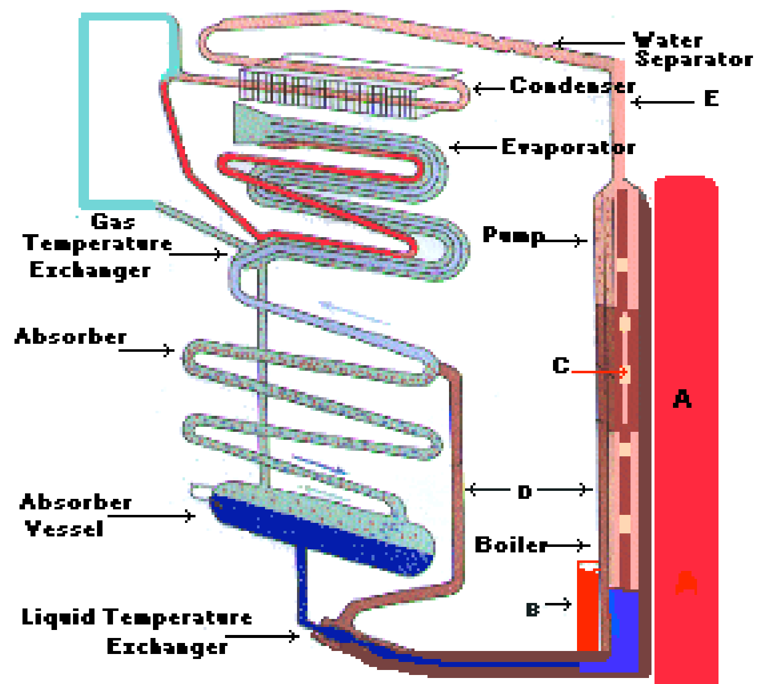


Leo Szillard

Coupling a flow from a hot bath to a cooler intermediate one to a flow from the cold bath to the intermediate one, heat is pumped from the cold bath.

$$\frac{\dot{Q}_h}{T_h} + \frac{\dot{Q}_d}{T_d} + \frac{\dot{Q}_c}{T_c} > 0$$

$$\Delta S_h + \Delta S_c + \Delta S_d > 0$$



Continuous-cycle Absorbtion System

No moving parts

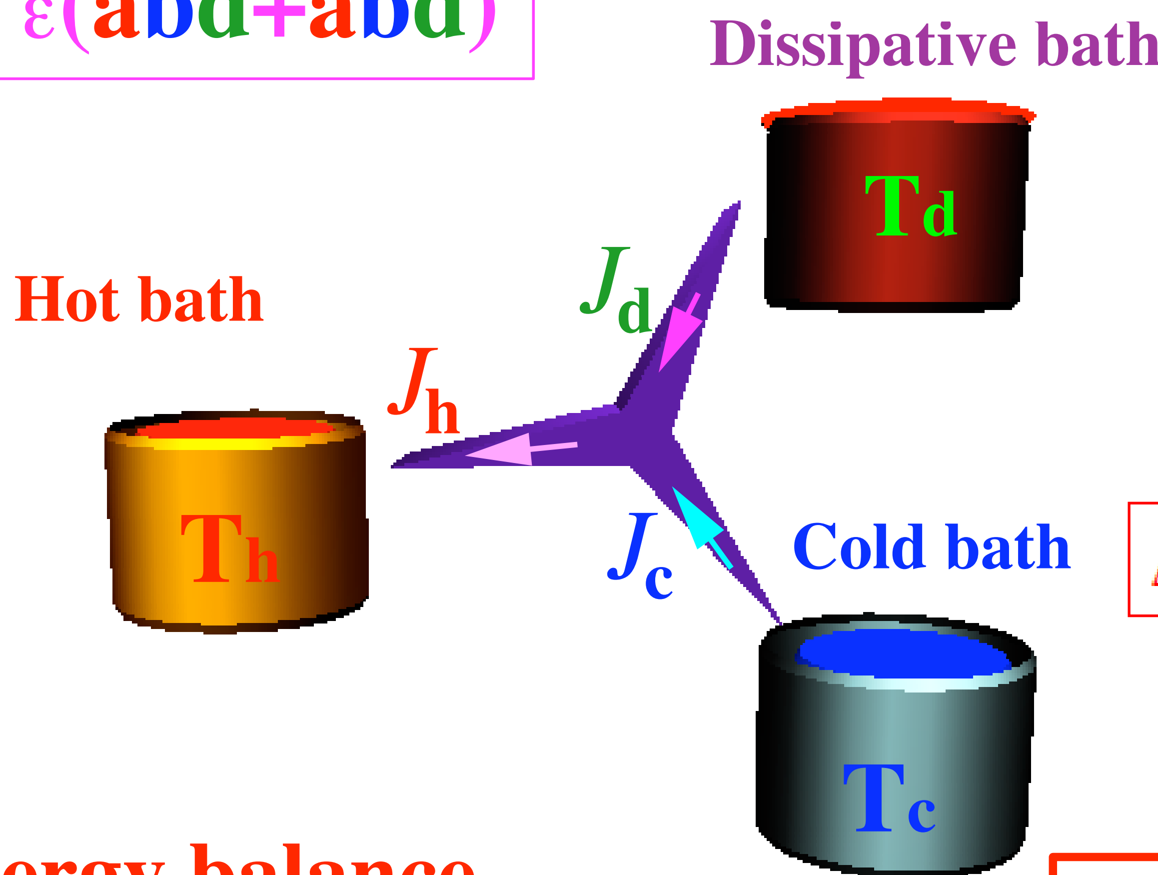
The quantum trickle

$$H = H_0 + H_{int}$$

$$H_0 = \omega_h a^\dagger a + \omega_c b^\dagger b + \omega_d d^\dagger d$$

$$H_{int} = \varepsilon(a^\dagger b d + a b^\dagger d^\dagger)$$

Levi & Kosloff, PRL 108, 070604 (2012)



$$\Delta S_h + \Delta S_c + \Delta S_d > 0$$

Entropy production

Energy balance

$$\dot{J}_h + \dot{J}_c + \dot{J}_d = 0$$

$$\frac{J_h}{T_h} + \frac{J_c}{T_c} + \frac{J_d}{T_d} \geq 0$$

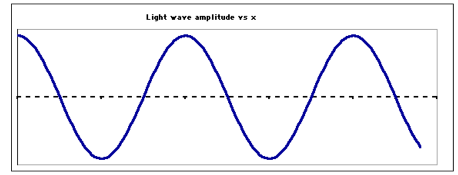
The quantum trickle semiclassical limit

$$H = H_0 + H_{int}$$

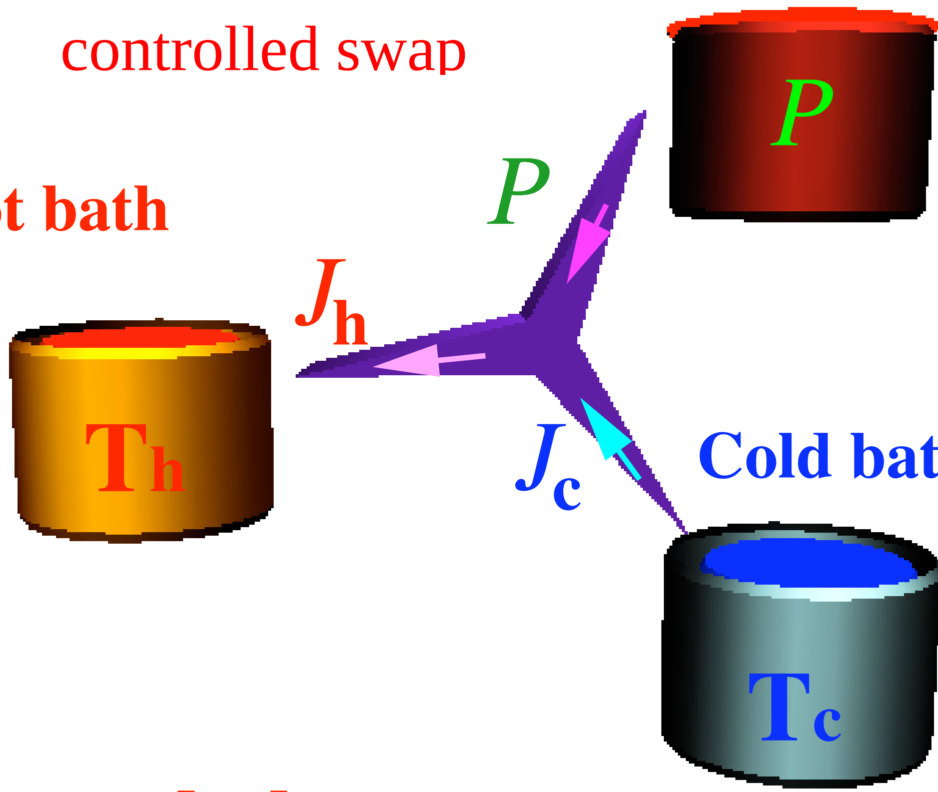
$$H_0 = \omega_h a^\dagger a + \omega_c b^\dagger b$$

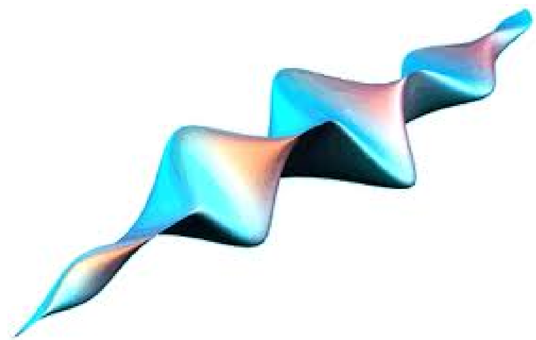
$$H_{int} = \varepsilon\left(a^\dagger b e^{-i\nu t} + a b^\dagger e^{+i\nu t}\right)$$

$$d \Rightarrow q e^{-i\nu t}$$



controlled swap





Energy balance

$$J_h + J_c + P = 0$$

Entropy production

$$\frac{J_h}{T_h} + \frac{J_c}{T_c} \geq 0$$

The quantum trickle semiclassical limit

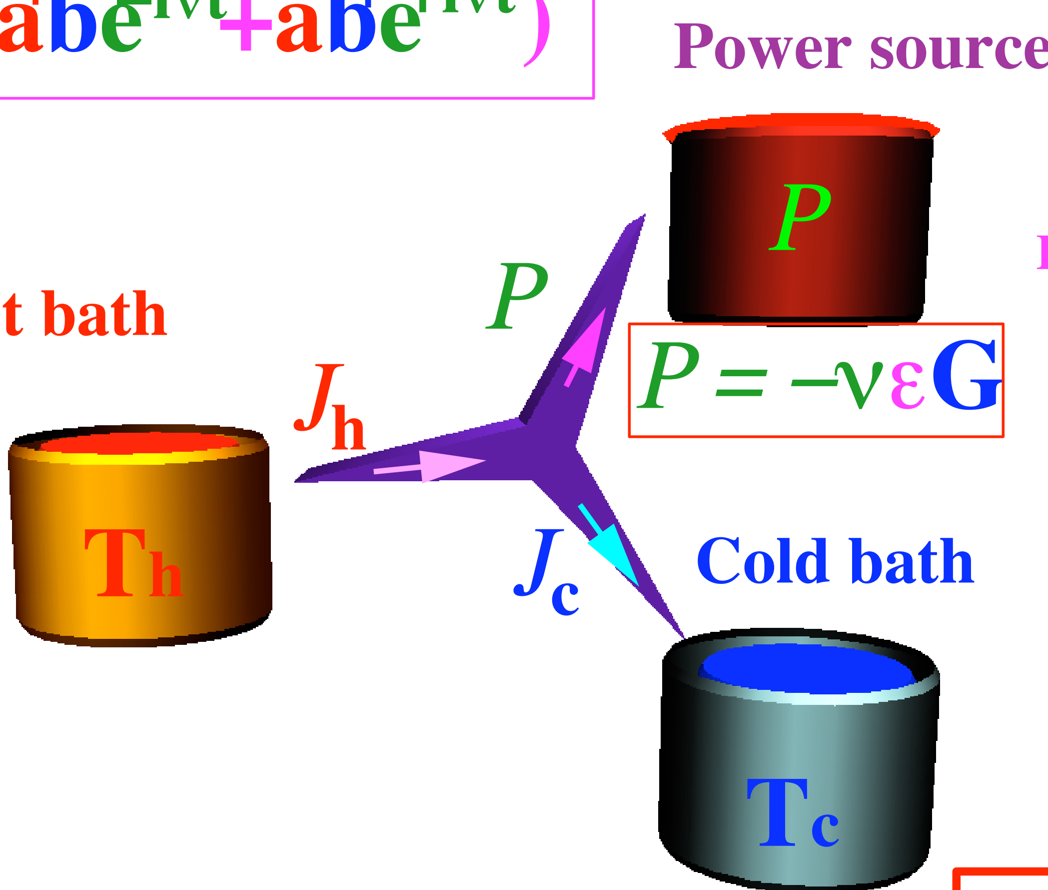
$$H = H_0 + H_{int}$$

As an Engine

$$H_0 = \omega_h a^\dagger a + \omega_c b^\dagger b$$

$$H_{int} = \varepsilon\left(a^\dagger b e^{-i\nu t} + a b^\dagger e^{+i\nu t}\right)$$

$$d \Rightarrow q e^{-i\nu t}$$



$$\eta = 1 - \sqrt{\frac{T_c}{T_h}}$$

Efficiency at Maximum power

R.K. JCP 80 1625 (1984)

Entropy production

$$\frac{J_h}{T_h} + \frac{J_c}{T_c} \geq 0$$

Energy balance

$$J_h + J_c + P = 0$$

The quantum trickle *absorption refrigerator*

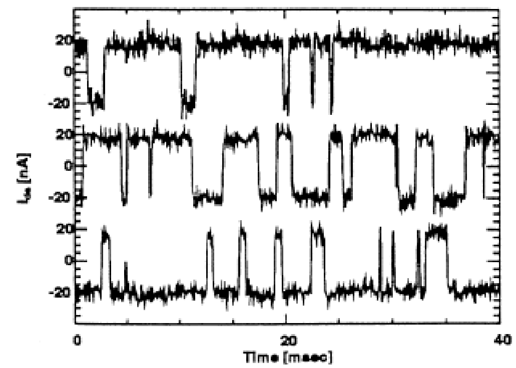
$$H = H_0 + H_{int}$$

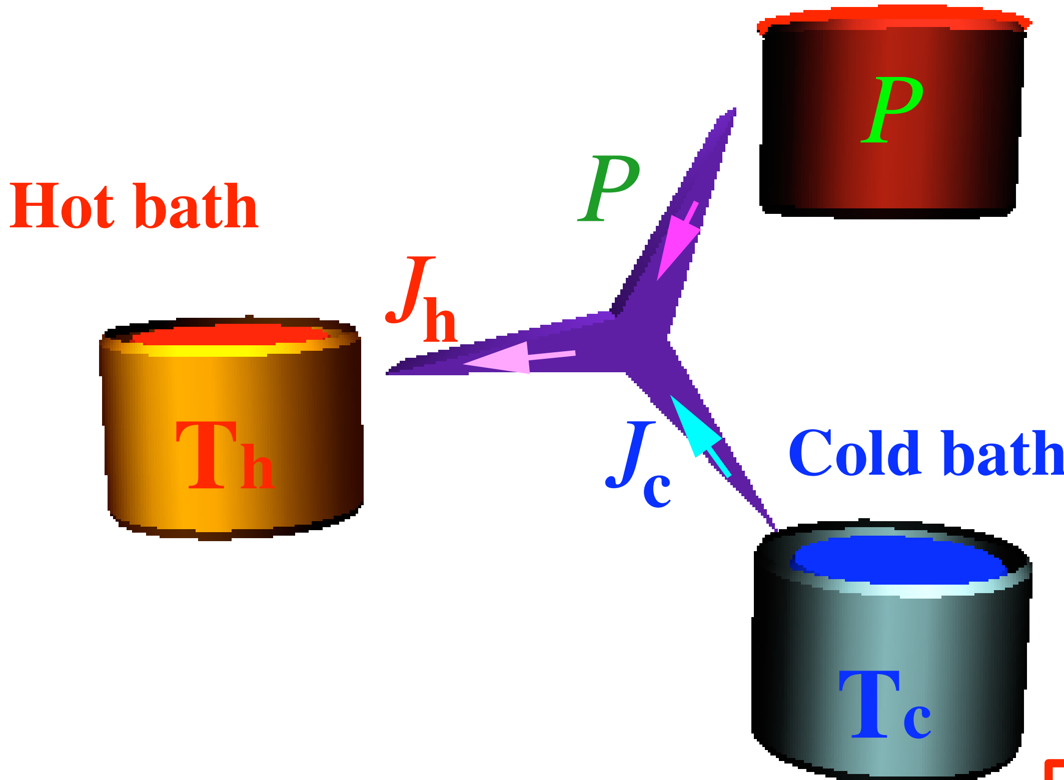
$$H_0 = \omega_h a^\dagger a + \omega_c b^\dagger b$$

$$H_{int} = f(t)(a^\dagger b + a b^\dagger)$$

$f(t)$ noise field

Power source





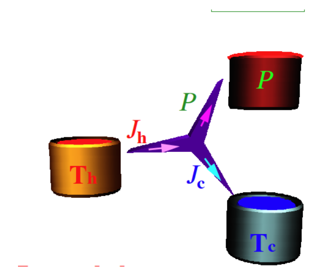
Entropy production

Energy balance

$$J_h + J_c + P = 0$$

$$\boxed{\frac{J_h}{T_h} + \frac{J_c}{T_c} \geq 0}$$

The quantum trickle *absorption refrigerator*



$$H_0 = \omega_h a^\dagger a + \omega_c b^\dagger b$$

$$H = H_0 + H_{int}$$

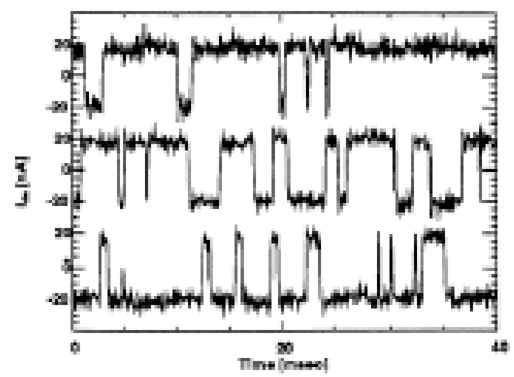
Equations of Motion

$$H_{int} = f(t)(a^\dagger b + a b^\dagger) = f(t)X$$

$f(t)$ Guassian noise $\langle f(t)f(t')\rangle = 2\eta\delta(t-t')$ then:

$$\dot{O} = +i[H_0, O] + \mathcal{L}_n(O) + \mathcal{L}_c(O) + \mathcal{L}_h(O)$$

$$\mathcal{L}_n(O) = -\eta[X,[X,O]]$$



$$\mathcal{L}_c(O) = \Gamma_c(N_c+1)(b^\dagger O b - 1/2\{b^\dagger b, O\}) + \Gamma_c N_c (b O b^\dagger - 1/2\{b b^\dagger, O\})$$

$$\mathcal{L}_h(O) = \Gamma_h(N_h+1)(a^\dagger O a - 1/2\{a^\dagger a, O\}) + \Gamma_h N_h (a O a^\dagger - 1/2\{a a^\dagger, O\})$$

The quantum trickle *absorption refrigerator*

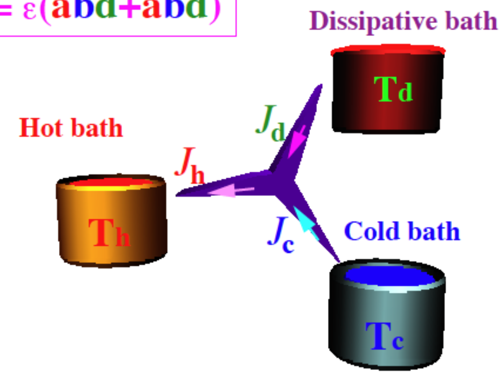
arXiv:1109.0728v1

$$H = H_0 + H_{int}$$

$$H_0 = \omega_h a^\dagger a + \omega_c b^\dagger b + \omega_d d^\dagger d$$

$$H_{int} = \varepsilon(a^\dagger b d + a b^\dagger d^\dagger)$$

Heat driven absorption refrigerator:



$$\boxed{\dot{O} = +i[H_0,O] + L_d(O) + L_c(O) + L_h(O)}$$

$$L_d(O) = \Gamma_d(N_d+1)\left(ab^\dagger O ab^\dagger - 1/2\{ab^\dagger ab^\dagger, O\}\right) + \Gamma_d N_d\left(ba^\dagger O ba^\dagger - 1/2\{ba^\dagger ba^\dagger, O\}\right)$$

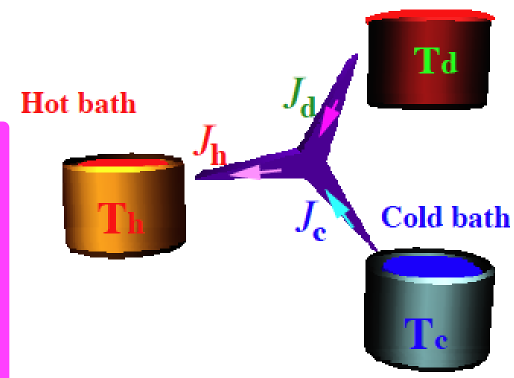
In the hight temeperature limit $T_d \rightarrow \infty$

$$L_d(O) = -\eta\left([X,[X,O]] + [Y,[Y,O]]\right)$$

Guassian noise is equivalent to the high temperature limit.

The Guassian generator can be generated by quantum measurement.

The quantum trickle *absorption refrigerator*



G= Nh−Nc ≥ 0

$$J_c = \hbar\omega_c \Gamma_d \frac{N_c - N_h}{1 + \Gamma_d\left(N_c \Gamma_c^{-1} e^{\frac{\hbar\omega_c}{kT_c}} + N_h \Gamma_h^{-1} e^{\frac{\hbar\omega_h}{kT_h}}\right)}$$

$$N = \left(e^{\frac{\hbar\omega}{kT}} \mp 1\right)^{-1}$$

$$J_c \geq 0 \rightarrow \frac{\omega_c}{\omega_h} \leq \frac{T_c}{T_h}$$

$$COP = \frac{J_c}{J_d} = \frac{\omega_c}{\omega_h - \omega_c} \leq \frac{T_c}{T_h - T_c}$$

Otto cycle cop

limited by Carnot

All types of refrigerators have universal properties as $T_c \rightarrow 0$.

In the power driven refrigerators the cold current becomes:

$$\mathcal{J}_c \approx \hbar\omega_c^- \frac{2\epsilon^2 \bar{\Gamma}}{4\epsilon^2 + \Gamma_c \Gamma_h} \cdot G \ , \quad \textit{where the gain } G = N_c^- - N_h^-$$

and $\bar{\Gamma} = \frac{\Gamma_c \Gamma_h}{\Gamma_c + \Gamma_h}$.

In the 3-level absorption refrigerator:

$$\mathcal{J}_c \ = \ \hbar\omega_c \frac{\Gamma_c \Gamma_h \Gamma_w}{\Delta} \cdot G \quad \textit{where } G = e^{-\frac{\hbar\omega_w}{k_B T_w}} e^{-\frac{\hbar\omega_c}{k_B T_c}} - e^{-\frac{\hbar\omega_h}{k_B T_h}}$$

.

In the Guassian noise driven refrigerator:

$$\mathcal{J}_c \ = \ \hbar\omega_c \frac{2\eta\bar{\Gamma}}{2\eta + \bar{\Gamma}} \cdot G \qquad \textit{where} \quad G = N_c - N_h$$

In the Poisson driven refrigerator:

$$\mathcal{J}_c \approx \hbar\Omega_- \frac{2\eta\bar{\Gamma}}{2\eta + \bar{\bar{\Gamma}}} \cdot G \ , \quad \textit{where } G = (N_c^- - N_h^+) \tag{3}$$

and $\Omega_- \approx \omega_c - \frac{\epsilon^2}{\omega_h - \omega_c}$.

Optimising the gain G is when $\omega_h \to \infty$, therefore $G \sim e^{-\frac{\hbar\omega_c}{k_B T_c}}$.

.

The universal optimised cooling current as $T_c \to 0$ becomes:

$$\boxed{\mathscr{J}_c \ = \ \hbar\omega_c \cdot \gamma_c \cdot e^{-\frac{\hbar\omega_c}{k_B T_c}}\ .}$$

Further optimisation with respect to ω_c is dominated by the exponential Boltzmann factor

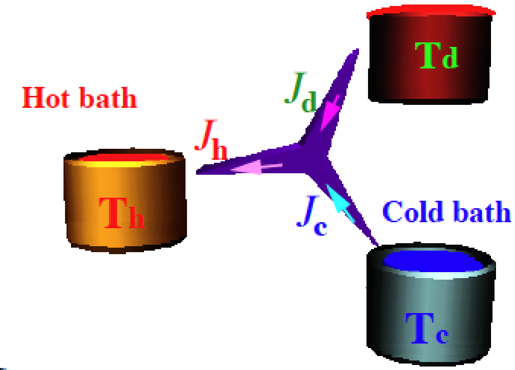
As a result

$$\boxed{\omega_c^* \propto T_c}$$

obtaining: $\mathscr{J}_c \propto \omega_c^* \cdot \gamma_c(\omega_c^*)$.

$\omega_c \propto T_c$ allows to translate the temperature scaling relations to the low frequency scaling relations of $\gamma_c(\omega) \sim \omega^\mu$ and $c_V(\omega) \sim \omega^\eta$ when $\omega \to 0$.

The quantum trickle *absorption refrigerator*

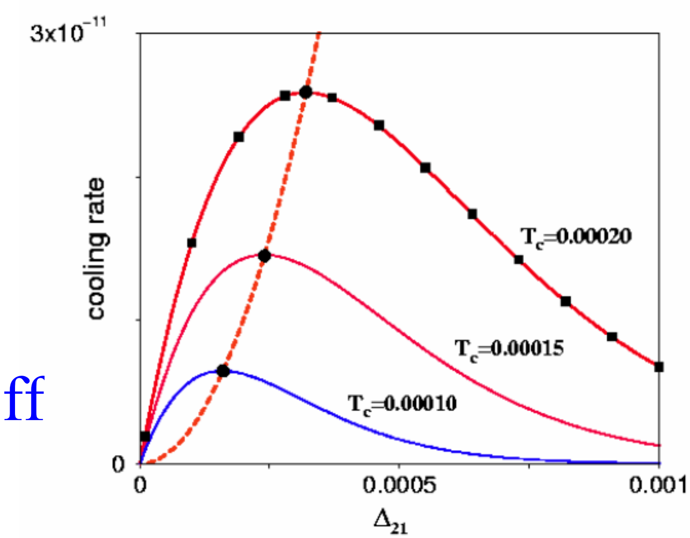


Optimizing the cooling rate when $\mathbf{T_c} \to 0$.

$$\boxed{\mathbf{J_c} \sim \hbar\omega_c \, \Gamma_c \, e^{-\frac{\hbar\omega_c}{kT_c}}}$$

with the result $\omega_c \propto \mathbf{T_c}$.

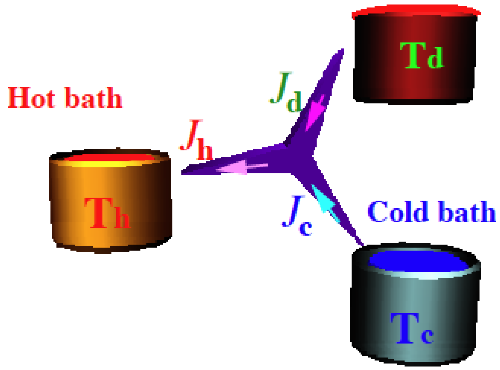
The optimal cooling rate



Amikam Levy, Robert Alicki, Ronnie Kosloff

Phys Rev E 85 013420 (2012).

The quantum trickle *absorption refrigerator*

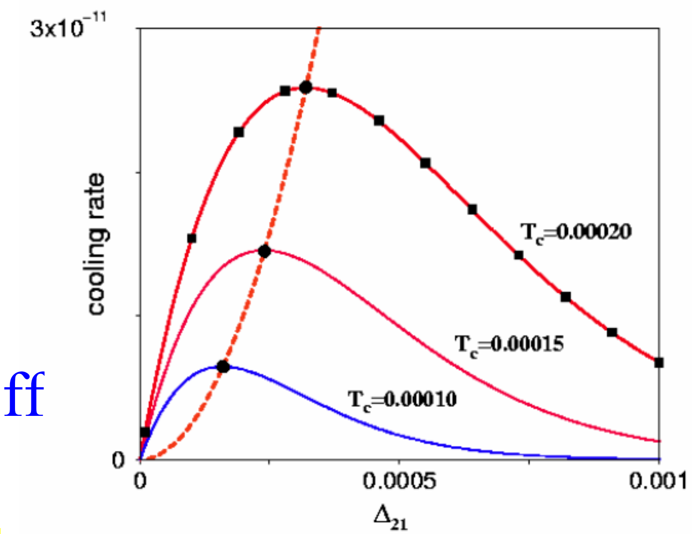


Optimizing the cooling rate when $\mathbf{T_c} \to 0$.

$$\boxed{\mathbf{J_c} \sim \hbar\omega_c \Gamma_c e^{-\frac{\hbar\omega_c}{kT_c}}}$$

with the result $\omega_c \propto \mathbf{T_c}$.

The optimal cooling rate

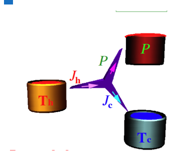


Amikam Levy, Robert Alicki, Ronnie Kosloff

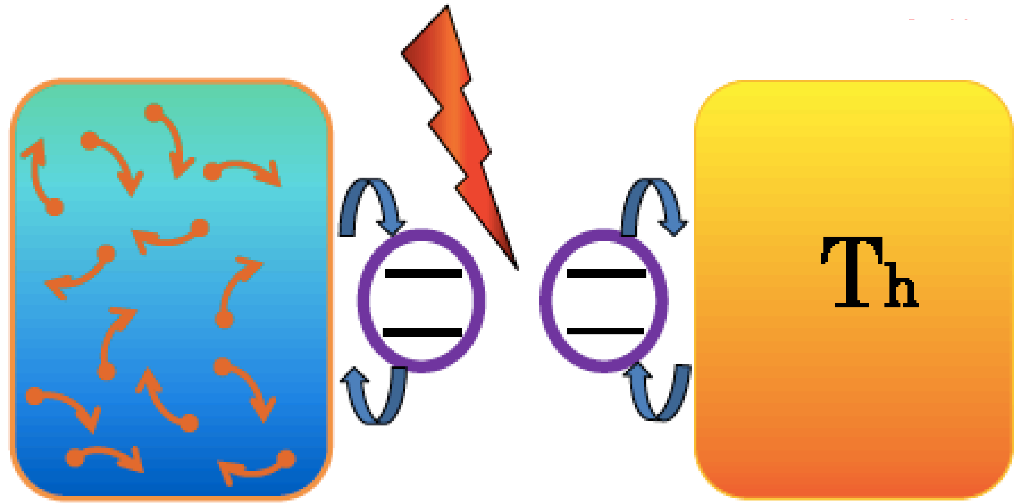
Phys Rev E 85 013420 (2012).

The quantum trickle *The III-law of Thermodynamics*

Ideal Bose/Fermi gas cold heat bath



Cooling occurs from non-elastic scattering of the gas from a heavy particle with internal structure approximated as TLS.



Low density limit quantum master equation

Maxwell distribution

Relaxation rate $\gamma_c = 2\pi n \int d^3\vec{p} \int d^3\vec{p}'\, \delta(E(\vec{p}') - E(\vec{p}) - \hbar\omega_c)\, f_{T_c}(\vec{p}) \left|T(\vec{p}',\vec{p})\right|^2$

Particles density

Transition matrix

$$\left|T(\vec{p}',\vec{p})\right|^2 = \left(\frac{4\pi a_s}{m}\right)^2$$

At low temperature

$$\gamma_c = (4\pi)^4 \left(\frac{\beta_c}{2\pi m}\right)^{1/2} a_s^2 n \omega_c K_1\left(\frac{\beta_c \omega_c}{2}\right) e^{\frac{\beta_c \omega_c}{2}}$$

To fulfil "Nernst's heat theorem" the scaling of the relaxation rate is restricted to $\gamma_c(\omega) \sim \omega^{\alpha}$ and $\alpha > 0$.

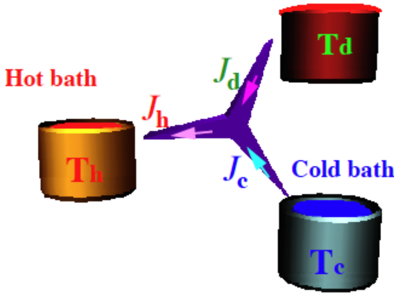
The fulfilment of the unattainability principle depends on the ratio between the relaxation rate and the heat capacity $\gamma_c / c_V \sim \omega^{\zeta - 1}$ where $\zeta > 1$.

For three-dimentional ideal degenerate Bose gases $c_V \propto T_c^{3/2}$.
For degenerate Fermi gas $c_V \propto T_c$.

In both cases the fraction of the gas that can be cooled decreases with temperature. Based on a collision model when cooling occurs due to inelastic scattering the scaling exponent of the III-law becomes:

$$\boxed{\zeta = 3/2}$$

The quantum trickle *The III-law of Thermodynamics*

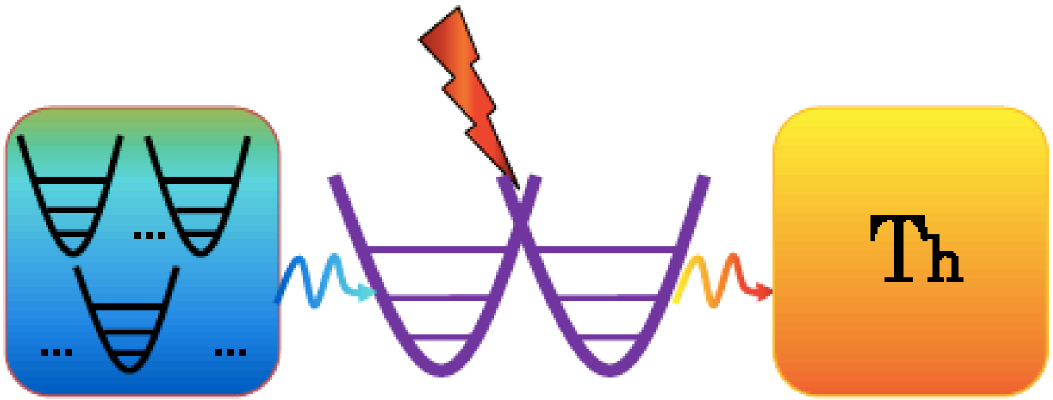


Harmonic cold heat bath

- Electromagnetic field
- Acoustic phonon

$$H_B = \sum_k \omega(k) c^\dagger(k) c(k)$$

$$H_{int} = (b + b^\dagger) \sum_k g(k) c(k) + \bar{g}(k) c^\dagger(k)$$



Relaxation rate: $\gamma_c = \pi \sum_k |g(k)|^2 \delta(\omega(k) - \omega_c)[1 - e^{-\omega(k)\beta_c}]^{-1}$

For the Bosonic field in d-dimensional space :

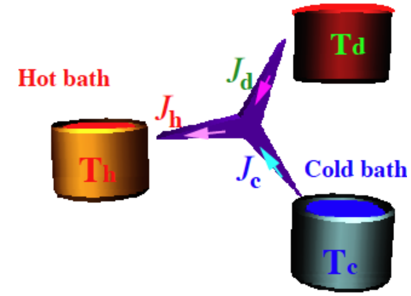
$$\boxed{\gamma_c \sim \omega_c^\kappa \omega_c^{d-1} [1 - e^{-\omega_c \beta_c}]^{-1}}$$

ω_c^κ - Scaling of the coupling strength

ω_c^{d-1} - Scaling of the density of modes

A. Levy and R. Kosloff, PRL 108, 070604 (2012)

The quantum trickle *The III-law of Thermodynamics*



The final current scaling $\boxed{\mathcal{J}_c^{opt} \sim T_c^{d+\kappa}} \longrightarrow \underline{\alpha = d+\kappa > 1}$

NHT

The heat capacity scaling $\boxed{C_V(T_c) \sim T_c^{d}}$

The rate of temperature decrease scaling $\boxed{\dfrac{dT_c}{dt} \sim -(T_c)^{\kappa}}$

Acoustic phonons:

Dispersion law: $\omega(k) = v|k|$

Form factor : $g(k) = |k| / \sqrt{\omega(k)}$

$\kappa = 1$

$\underline{\zeta = \kappa \geq 1}$

UP

The condition $\kappa \geq 1$ exclude exotic dispersion law $\omega(k) = |k|^{\sigma}$ with $\sigma < 1$

The quantum trickle *The III–law of Thermodynamics*

The quest to cool to the absolute zero temperature

3D phonon bath ***the existance of ground state.***

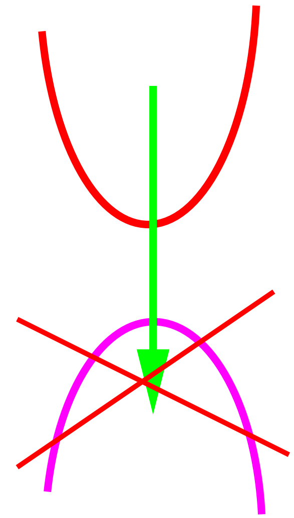
Conditions on the spectral density function $g(\omega)$

$$H = \sum \{\omega(k) a^{+}(k)\, a(k) + [g(k) a(k) + g^{*}(k) a^{+}(k)]\}$$

$$a(k) \rightarrow b(k) = a(k) + g(k)/\omega(k)$$

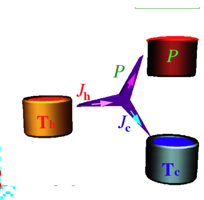
$$H = \sum \{\omega(k) b^{+}(k)\, b(k)\} - E_0 \quad , E_0 = \sum g(k)^2/\omega(k)$$

$$E_0 = \sum g(k)^2/\omega(k) < \infty$$



$$\boxed{\sum g(k)^2/\omega^2(k) < \infty} \qquad |g(\omega)|^2 \sim \omega^k \qquad k > 2-d$$

The quantum trickle *The III–law of Thermodynamics*



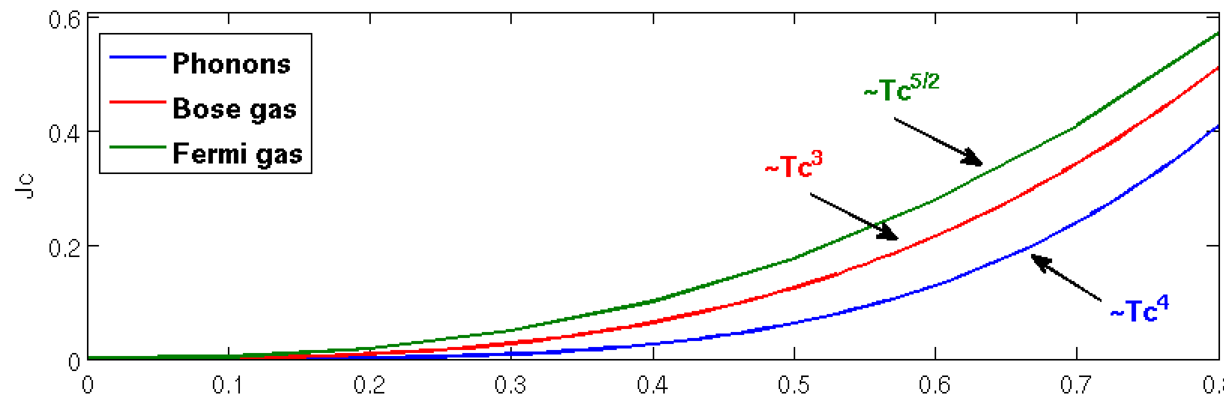
The quest to cool to the absolute zero temperature

3D phonon *vs. Bos Gas heat bath*

3D–Phonon

$$\boxed{J_c \sim -T_c^4}$$

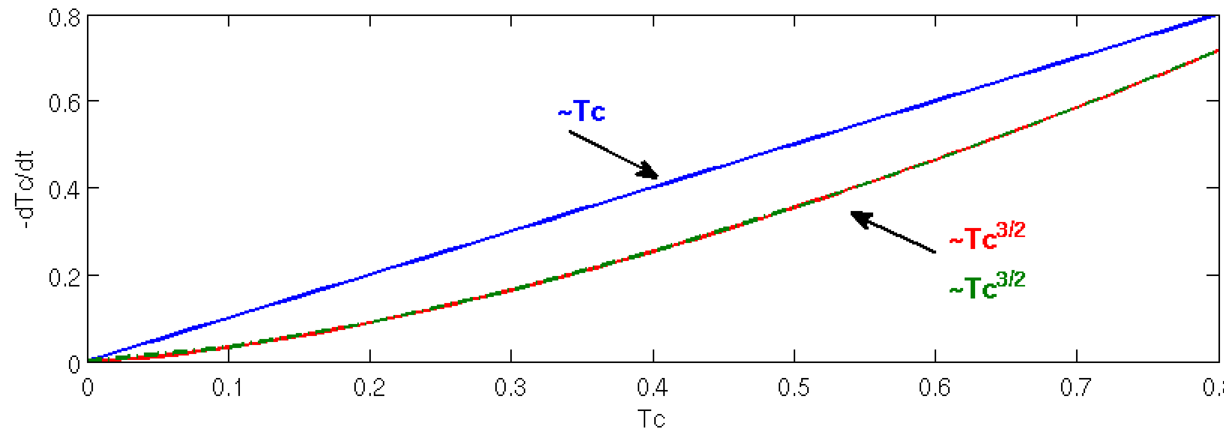
$$\boxed{\frac{dT_c}{dt} \sim -T_c^1}$$



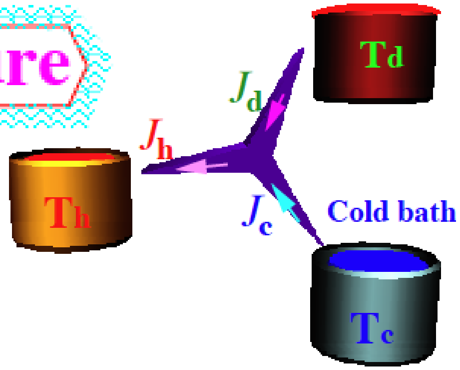
Bose gas

$$\boxed{J_c \sim -T_c^3}$$

$$\boxed{\frac{dT_c}{dt} \sim -T_c^{3/2}}$$



The quest to cool to the absolute zero temperature



Universal optimization

$$\boxed{\bar{\mathbf{J}}_c \propto \hbar\omega_c \cdot \Gamma_c(\omega_c, T_c) \cdot (\mathbf{N}_c - \mathbf{N}_h)}$$

quant *coupling* *gain*

$\omega_c \propto T_c$ $\Gamma_c \propto T_c{}^{\kappa+d-1}$ *constant*

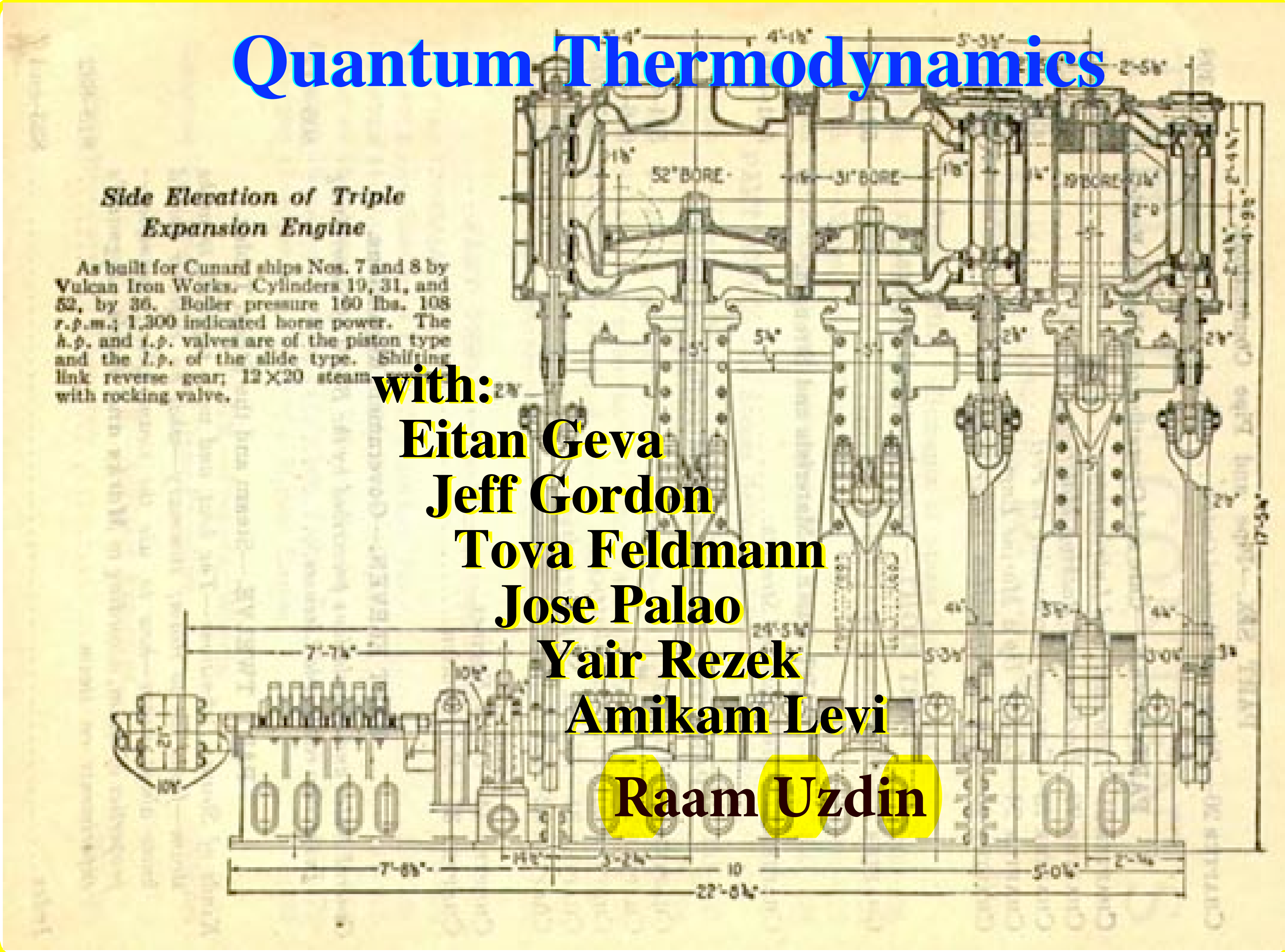
$$\boxed{\frac{dT_c(t)}{dt} = -c\, T_c^{\zeta}, \quad T_c \to 0 \;.\zeta > 1}$$

$$\boxed{\Delta \dot{S}_c \sim -T_c^{\alpha} \quad , \quad \alpha > 0 \;.}$$

$\zeta = \frac{3}{2}$ for cold Bose/Fermi gas. $\zeta = 1$ harmonic bath.

$\alpha = 2$ for Bose gas $\alpha = \frac{5}{2}$ for Fermi gas $\alpha = 3$ for harmonic bath.

Quantum Thermodynamics



with:

Eitan Geva

Jeff Gordon

Tova Feldmann

Jose Palao

Yair Rezek

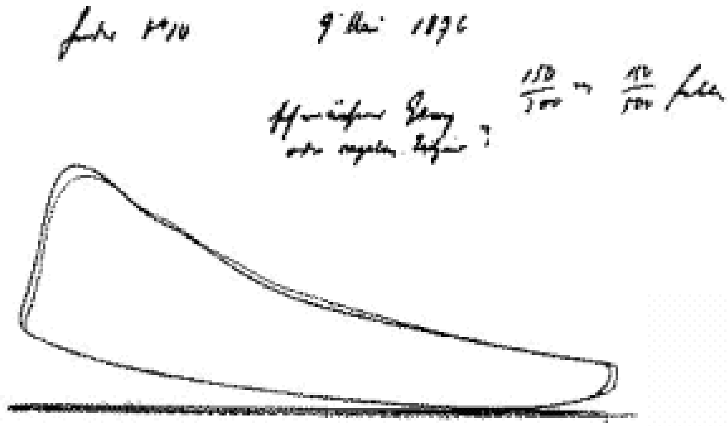
Amikam Levi

Raam Uzdin

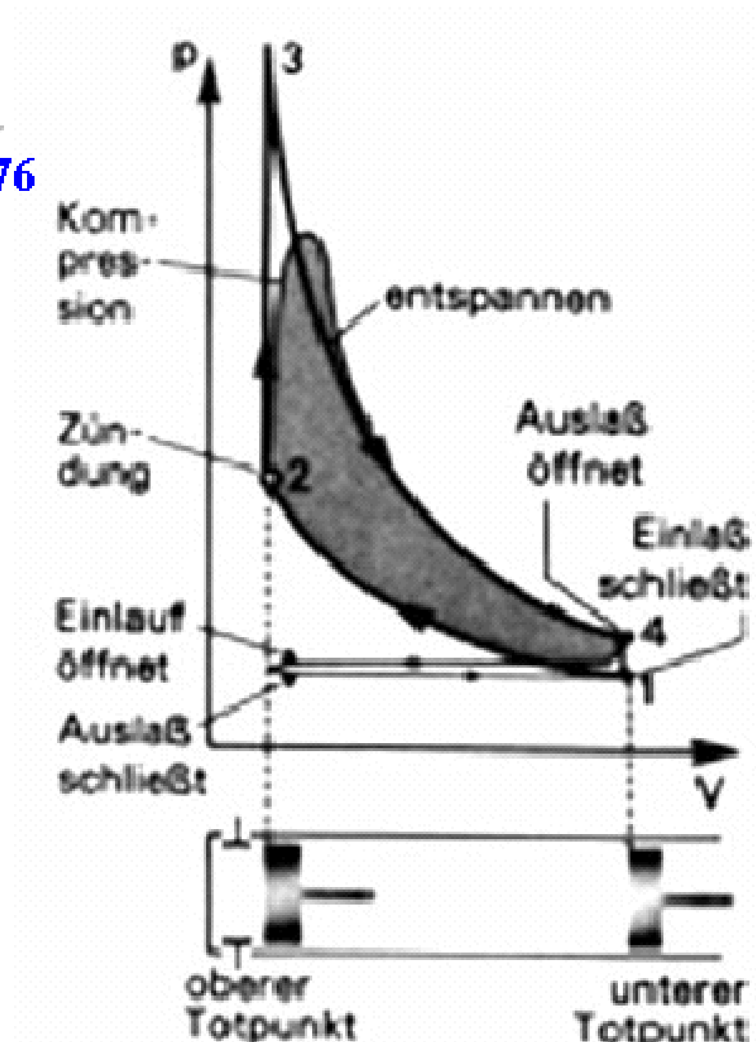
The end

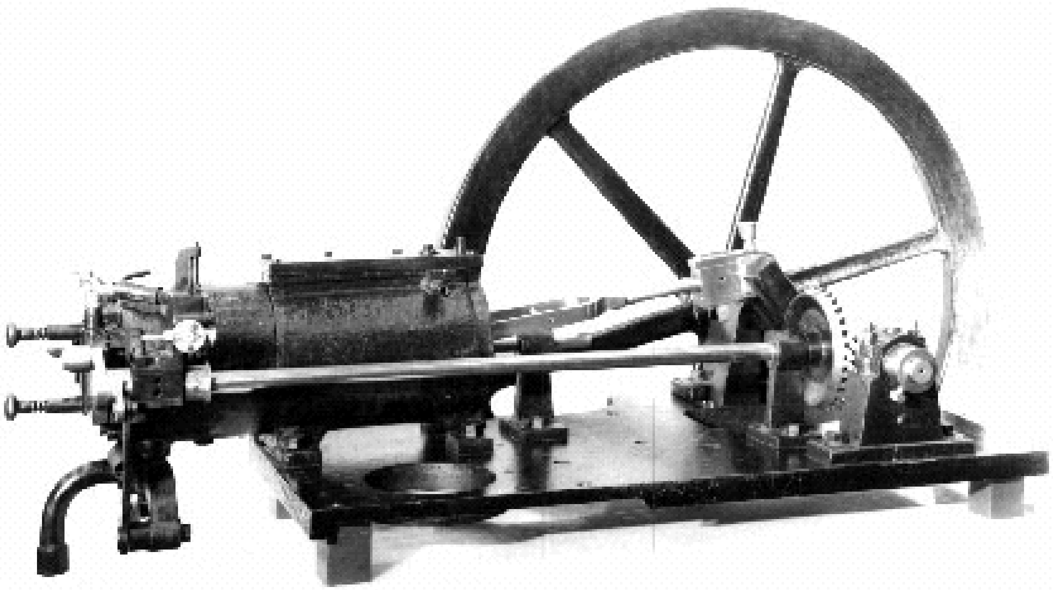
Thank you

Nicolaus August Otto



working diagram : 9th May 1876





four–stroke cycle engine 1876